

Macarro, Luis Narváez; Sevenheck, Christian**Tautological systems and free divisors.** (English) Zbl 1451.14054

Adv. Math. 352, 372-405 (2019).

The paper under review deals with systems of differential equations defined by certain prehomogeneous vector spaces endowed with actions of algebraic groups admitting open dense orbits. Such \mathcal{D} -modules can be considered as examples of the so-called tautological systems studied in many works (see [*M. Kapranov*, in: Integrable systems and algebraic geometry. Proceedings of the 41st Taniguchi symposium, Kobe, Japan, June 30–July 4, 1997, and in Kyoto, Japan, July 7–11 1997. Singapore: World Scientific. 236–281 (1998; [Zbl 0987.33008](#)); *B. H. Lian* et al., J. Eur. Math. Soc. (JEMS) 15, No. 4, 1457–1483 (2013; [Zbl 1272.14033](#))]).

In fact, the authors investigate the case of reductive groups whose orbits have complements which are linear free divisors satisfying the strongly Koszul condition [*M. Granger* and *M. Schulze*, Publ. Res. Inst. Math. Sci. 46, No. 3, 479–506 (2010; [Zbl 1202.14046](#))]. Under these assumptions it is proved that the associated tautological systems underlie mixed Hodge modules. Moreover, the authors give an explicit representation of the corresponding \mathcal{D} -modules similarly to the case of GKZ-systems [*T. Reichelt*, Compos. Math. 150, No. 6, 911–941 (2014; [Zbl 1315.14016](#))].

Reviewer: [Aleksandr G. Aleksandrov \(Moskva\)](#)**MSC:**

- [14F10](#) Differentials and other special sheaves; D -modules; Bernstein-Sato ideals and polynomials
- [32S40](#) Monodromy; relations with differential equations and D -modules (complex-analytic aspects)
- [32S35](#) Mixed Hodge theory of singular varieties (complex-analytic aspects)
- [14M17](#) Homogeneous spaces and generalizations

Keywords:

tautological systems; GKZ-systems; quantum D -modules; toric mirror symmetry; mixed Hodge modules; linear free divisors; Fourier-Laplace transforms; Radon transformation; Spencer complexes; Lie-Rinehart-algebras; Landau-Ginzburg models

Full Text: [DOI](#) [arXiv](#)**References:**

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