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Generalised primal-dual grids for unstructured co-volume schemes. (English) Zbl 1416.65325

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Summary: The generation of high-quality staggered unstructured grids is considered, leading to the development of a new optimisation-based strategy designed to construct weighted 'regular-power' tessellations appropriate for co-volume type numerical discretisation techniques. This new framework aims to extend the conventional Delaunay-Voronoi primal-dual structure; seeking to assemble generalised orthogonal tessellations with enhanced geometric quality. The construction of these grids is motivated by the desire to improve the performance and accuracy of numerical methods based on unstructured co-volume type schemes, including various staggered grid techniques for the simulation of fluid dynamics and hyperbolic transport. In this study, a new hybrid optimisation strategy is proposed; seeking to optimise the geometry, topology and weights associated with general, two-dimensional regular-power tessellations using a combination of gradient-ascent and energy-based techniques. The performance of this new method is tested experimentally, with a range of complex, multi-resolution primal-dual grids generated for various coastal and regional ocean modelling applications.

MSC:

65M50 Mesh generation, refinement, and adaptive methods for the numerical solution of initial value and initial-boundary value problems involving PDEs
35L40 First-order hyperbolic systems
65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

Keywords:

mesh generation; mesh optimization; primal-dual pairs; power diagrams; Voronoi diagrams; covolume discretization

Software:

2D triangulations; CGAL; CGALmesh; Gmsh; MPAS-Ocean; Netgen; TetGen; Triangle

Full Text: DOI

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