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New perspectives of granular computing in relation geometry induced by pairings. (English)

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Summary: Given an arbitrary set Ω, we call a triple $\mathfrak{P} = (U, F, \Lambda)$, where U and Λ are two non-empty sets and F is a map from $U \times \Omega$ into Λ, a *pairing* on Ω. A pairing is an abstract mathematical generalization of the notion of *information table*, classically used in several scopes of granular computing and rough set theory. In this paper we undertake the study of pairings in relation to specific types of set operators, set systems and binary relations appearing in several branches of pure mathematics and information sciences. For example, an intersection-closed system $MAXP(\mathfrak{P})$ on Ω can be canonically associated with any pairing $\mathfrak{P}$ on Ω and we showed that for any intersection-closed system $\mathfrak{S}$ on an arbitrary (even infinite) set Ω there exists a pairing $\mathfrak{P}$ on Ω such that $MAXP(\mathfrak{P}) = \mathfrak{S}$. Next, we introduce some classes of pairings whose properties have a close analogy with corresponding notions derived from topology and matroid theory. We describe such classifications by means of a binary relation $\leftarrow_{\mathfrak{P}}$ on the power set $\mathcal{P}(\Omega)$ canonically associated with any pairing $\mathfrak{P}$. Using such a relation, we analyze new properties of intersection-closed systems and related operators, both within concrete models induced by metric spaces and also in connection with basic notions of common interest in several scopes of pure and applied mathematics and information sciences.

MSC:

06A15	Galois correspondences, closure operators (in relation to ordered sets)
08A02	Relational systems, laws of composition
68P01	General topics in the theory of data
68P05	Data structures
54E35	Metric spaces, metrizability

Keywords:

pairings; closure operators; abstract simplicial complexes; information tables; metric spaces; granular computing

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