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Classification of Mukai pairs with corank 3. (Classification des paires de Mukai de corang 3.) (English. French summary) [Zbl 1419.14066](#)

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In this paper, the author studies Mukai pairs (X, \mathcal{E}) as introduced in [*S. Mukai*, “Problems on characterization of complex projective space”, in: *Birational Geometry of Algebraic Varieties, Open Problems, Katata, the 23rd Int’l Symp., Taniguchi Foundation. 57–60 (1988)*]. A Mukai pair (X, \mathcal{E}) consists of a smooth Fano n -fold X and an ample vector bundle \mathcal{E} on X of rank r with $c_1(X) = c_1(\mathcal{E})$. The corank of a Mukai pair (X, \mathcal{E}) of dimension n and rank r is defined as the integer $c = n - r + 1$. The cases $c = 0$ and 1 are treated in [*T. Fujita*, *Lect. Notes Math.* 1507, 105–112 (1992; [Zbl 0782.14018](#))]; [*T. Peternell*, *Int. J. Math.* 2, No. 3, 311–322 (1991; [Zbl 0744.14009](#))]; [*Y.-G. Ye* and *Q. Zhang*, *Duke Math. J.* 60, No. 3, 671–687 (1990; [Zbl 0709.14011](#))] and for the case $c = 2$ see [*T. Peternell* et al., *Math. Ann.* 294, No. 1, 151–165 (1992; [Zbl 0786.14027](#))]. In the present paper the author extends the above results to the case $c = 3$ and gives a complete classification of such Mukai pairs with $n \geq 5$ and $r = n - 2$. Furthermore, as a corollary to his main theorem, he points out that given (X, \mathcal{E}) a generalized polarized pair of dimension $n \geq 5$ and rank $n - 2$, if there exists a $K3$ surface in X which is the zero locus of a section of \mathcal{E} then (X, \mathcal{E}) is one of the pairs as stated in his main theorem.

Reviewer: [Aigli Papantonopoulou \(Ewing\)](#)

MSC:

[14J45](#) Fano varieties

[14J40](#) n -folds ($n > 4$)

[14J60](#) Vector bundles on surfaces and higher-dimensional varieties, and their moduli

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