

Aizenman, Michael; Duminil-Copin, Hugo; Tassion, Vincent; Warzel, Simone
Emergent planarity in two-dimensional Ising models with finite-range interactions. (English)
Zbl 1417.82022
Invent. Math. 216, No. 3, 661-743 (2019).

Summary: The known Pfaffian structure of the boundary spin correlations, and more generally order-disorder correlation functions, is given a new explanation through simple topological considerations within the model's random current representation. This perspective is then employed in the proof that the Pfaffian structure of boundary correlations emerges asymptotically at criticality in Ising models on \mathbb{Z}^2 with finite-range interactions. The analysis is enabled by new results on the stochastic geometry of the corresponding random currents. The proven statement establishes an aspect of universality, seen here in the emergence of fermionic structures in two dimensions beyond the solvable cases.

MSC:

- 82C20** Dynamic lattice systems (kinetic Ising, etc.) and systems on graphs in time-dependent statistical mechanics Cited in 3 Documents
- 82C22** Interacting particle systems in time-dependent statistical mechanics
- 82C44** Dynamics of disordered systems (random Ising systems, etc.) in time-dependent statistical mechanics
- 58A17** Pfaffian systems

Keywords:

Ising models; Pfaffian structure; order-disorder correlation

Full Text: DOI

References:

- [1] Aizenman, M., Geometric analysis of (φ^4) fields and Ising models, Commun. Math. Phys., 86, 1-48, (1982) · Zbl 0533.58034
- [2] Aizenman, M.; Barsky, DJ; Fernández, R., The phase transition in a general class of Ising-type models is sharp, J. Stat. Phys., 47, 343-374, (1987)
- [3] Aizenman, M.; Burchard, A., Hölder regularity and dimension bounds for random curves, Duke Math. J., 99, 419-453, (1999) · Zbl 0944.60022
- [4] Aizenman, M.; Chayes, JT; Chayes, L.; Newman, CM, Discontinuity of the magnetization in one-dimensional $(1/|x-y|^2)$ Ising and Potts models, J. Stat. Phys., 50, 1-40, (1988) · Zbl 1084.82514
- [5] Aizenman, M.; Duminil-Copin, H.; Sidoravicius, V., Random currents and continuity of Ising model's spontaneous magnetization, Commun. Math. Phys., 334, 719-742, (2015) · Zbl 1315.82004
- [6] Aizenman, M.; Fernández, R., On the critical behavior of the magnetization in high-dimensional Ising models, J. Stat. Phys., 44, 393-454, (1986) · Zbl 0629.60106
- [7] Aizenman, A.; Fernández, Roberto, Critical exponents for long-range interactions, Lett. Math. Phys., 16, 39-49, (1988) · Zbl 0658.60136
- [8] Aizenman, M.; Warzel, S., Kac-Ward formula and its extension to order-disorder correlators through a graph zeta function, J. Stat. Phys., 173, 1755-1778, (2018) · Zbl 1405.82006
- [9] Baxter, RJ, Solvable eight-vertex model on an arbitrary planar lattice, Philos. Trans. R. Soc. Lond. Ser. A, 289, 315-346, (1978)
- [10] Benoist, S., Hongler, C.: The scaling limit of critical Ising interfaces is CLE(3) (2016). arXiv:1604.06975
- [11] Burton, RM; Keane, M., Density and uniqueness in percolation, Commun. Math. Phys., 121, 501-505, (1989) · Zbl 0662.60113
- [12] Chelkak, D., Cimasoni, D., Kassel, A.: Revisiting the combinatorics of the 2D Ising model. Ann. Inst. Henri Poincaré D (2016) [\textbf{\(to appear\)}](#) · Zbl 1380.82017
- [13] Chelkak, D.; Duminil-Copin, H.; Hongler, C., Crossing probabilities in topological rectangles for the critical planar FK-Ising model, Electron. J. Probab., 5, 28, (2016) · Zbl 1341.60124
- [14] Chelkak, D.; Duminil-Copin, H.; Hongler, C.; Kemppainen, A.; Smirnov, S., Convergence of Ising interfaces to Schramm's SLE curves, C. R. Acad. Sci. Paris Math., 352, 157-161, (2014) · Zbl 1294.82007

- [15] Chelkak, D.; Smirnov, S., Universality in the 2D Ising model and conformal invariance of fermionic observables, *Invent. Math.*, 189, 515-580, (2012) · [Zbl 1257.82020](#)
- [16] Duminil-Copin, H.: Lectures on the Ising and Potts models on the hypercubic lattice (2017). [arXiv:1707.00520](#)
- [17] Duminil-Copin, H.; Hongler, C.; Nolin, P., Connection probabilities and RSW-type bounds for the two-dimensional FK Ising model, *Commun. Pure Appl. Math.*, 64, 1165-1198, (2011) · [Zbl 1227.82015](#)
- [18] Duminil-Copin, H., Lis, M.: On the double random current nesting field (2017). [arXiv:1712.02305](#)
- [19] Duminil-Copin, H.; Sidoravicius, V.; Tassion, V., Absence of infinite cluster for critical Bernoulli percolation on slabs, *Commun. Pure Appl. Math.*, 69, 1397-1411, (2016) · [Zbl 1342.82076](#)
- [20] Duminil-Copin, H.; Tassion, V., A new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model, *Commun. Math. Phys.*, 343, 725-745, (2016) · [Zbl 1342.82026](#)
- [21] Duminil-Copin, H., Tassion, V.: RSW and box-crossing property for planar percolation. In: *Proceedings of the International Congress of Mathematical Physics (2016)* · [Zbl 1342.82026](#)
- [22] Fisher, ME, On the dimer solution of planar Ising models, *J. Math. Phys.*, 7, 1776-1781, (1966)
- [23] Georgii, H.-O.: Gibbs measures and phase transitions, 2nd edn. volume 9 of *de Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin (2011) · [Zbl 1225.60001](#)
- [24] Giuliani, A.; Greenblatt, R.; Mastropietro, V., The scaling limit of the energy correlations in non-integrable Ising models, *J. Math. Phys.*, 53, 095214, (2012) · [Zbl 1278.82011](#)
- [25] Griffiths, R., Correlation in Ising ferromagnets I, II, *J. Math. Phys.*, 8, 478-489, (1967)
- [26] Groeneveld, J.; Boel, RJ; Kasteleyn, PW, Correlation-function identities for general planar Ising systems, *Physica*, 93A, 138-154, (1978)
- [27] Griffiths, RB; Hurst, CA; Sherman, S., Concavity of magnetization of an Ising ferromagnet in a positive external field, *J. Math. Phys.*, 11, 790-795, (1970)
- [28] Grimmett, G.: *The Random-Cluster Model*, *Grundlehren der Mathematischen Wissenschaften (Fundamental Principles of Mathematical Sciences)*, vol. 333. Springer, Berlin (2006) · [Zbl 1122.60087](#)
- [29] Grimmett, G.; Janson, S., Random even subgraphs, *Electron. J. Probab.*, 16, 1-19, (2009) · [Zbl 1214.05155](#)
- [30] Hegerfeldt, GC, Correlation inequalities for Ising ferromagnets with symmetries, *Commun. Math. Phys.*, 57, 259-266, (1977)
- [31] Hurst, CA; Green, HS, New solution of the Ising problem for a rectangular lattice, *J. Chem. Phys.*, 33, 1059-1062, (1960)
- [32] Kadanoff, LP, Spin-spin correlation in the two-dimensional Ising model, *Nuovo Cimento*, 44, 276-305, (1966)
- [33] Kadanoff, LP; Ceva, H., Determination of an operator algebra for the two-dimensional Ising model, *Phys. Rev. B*, 3, 3918-3939, (1971)
- [34] Kasteleyn, PW, Dimer statistics and phase transitions, *J. Math. Phys.*, 4, 287-293, (1963)
- [35] Kaufman, B., Crystal statistics. II. Partition function evaluated by spinor analysis, *Phys. Rev.*, 76, 1232-1243, (1949) · [Zbl 0035.42801](#)
- [36] Lieb, EH, A refinement of Simon's correlation inequality, *Commun. Math. Phys.*, 77, 127-135, (1980)
- [37] Lis, M., The planar Ising model and total positivity, *J. Stat. Phys.*, 166, 72-89, (2017) · [Zbl 1371.82024](#)
- [38] Lupu, T., Werner, W.: A note on Ising random currents, Ising-FK, loop-soups and the Gaussian free field. *Electron. Commun. Probab.* [\textbf{21}](#), paper no. 13, 7 (2016) · [Zbl 1338.60236](#)
- [39] Manolescu, I., Raoufi, A.: The phase transitions of the random-cluster and Potts models on slabs with $(q \geq 1)$ are sharp (2016). [arXiv:1604.01299](#)
- [40] McCoy, B.M., Wu, T.T.: *The Two-Dimensional Ising Model*. Harvard University Press, Cambridge (1973) · [Zbl 1094.82500](#)
- [41] McCoy, BM; Perk, JHH; Wu, TT, Ising field theory: quadratic difference equations for the (n) -point Green's functions on the square lattice, *Phys. Rev. Lett.*, 46, 757-760, (1981)
- [42] Messager, A.; Miracle-Sol, S., Correlation functions and boundary conditions in the Ising ferromagnet, *J. Stat. Phys.*, 17, 245-262, (1977)
- [43] Newman, CM; Tassion, V.; Wu, W., Critical percolation and the minimal spanning tree in slabs, *Commun. Pure Appl. Math.*, 70, 2084-2120, (2017) · [Zbl 1380.82025](#)
- [44] Pinson, H., Spencer, T.: Universality and the two-dimensional Ising model [\textbf{\(unpublished preprint\)}](#)
- [45] Onsager, L., Crystal statistics. I. A two-dimensional model with an order-disorder transition, *Phys. Rev.* (2), 65, 117-149, (1944) · [Zbl 0060.46001](#)
- [46] Russo, L., A note on percolation, *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete*, 43, 39-48, (1978) · [Zbl 0363.60120](#)
- [47] Schrader, R., New correlation inequalities for the Ising model and $(P(\phi))$ theories, *Phys. Rev. B*, 15, 2798, (1977)
- [48] Schultz, TD; Mattis, D.; Lieb, EH, Two-dimensional Ising model as a soluble problem of many fermions, *Rev. Mod. Phys.*, 36, 856, (1964)
- [49] Seymour, PD; Welsh, DJA, Percolation probabilities on the square lattice, *Ann. Discrete Math.*, 3, 227-245, (1978) · [Zbl 0405.60015](#)
- [50] Simon, B., Correlation inequalities and the decay of correlations in ferromagnets, *Commun. Math. Phys.*, 77, 111-126, (1980)
- [51] Tassion, V., Crossing probabilities for Voronoi percolation, *Ann. Probab.*, 44, 3385-3398, (2016) · [Zbl 1352.60130](#)

- [52] Waerden, BL, Die lange Reichweite der regelmässigen Atomanordnung in Mischkristallen, Z. Physik, 118, 473-488, (1941) · [Zbl 0026.28301](#)
- [53] Yang, CN, The spontaneous magnetization of a two-dimensional Ising model, Phys. Rev., 2, 808-816, (1952) · [Zbl 0046.45304](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.