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Mixing properties and central limit theorem for associated point processes. (English)

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Summary: Positively (resp. negatively) associated point processes are a class of point processes that induce attraction (resp. inhibition) between the points. As an important example, determinantal point processes (DPPs) are negatively associated. We prove α-mixing properties for associated spatial point processes by controlling their α-coefficients in terms of the first two intensity functions. A central limit theorem for functionals of associated point processes is deduced, using both the association and the α-mixing properties. We discuss in detail the case of DPPs, for which we obtain the limiting distribution of sums, over subsets of close enough points of the process, of any bounded function of the DPP. As an application, we get the asymptotic properties of the parametric two-step estimator of some inhomogeneous DPPs.

MSC:

60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
60F05 Central limit and other weak theorems

Keywords:

determinantal point process; negative association; parametric estimation; positive association; strong mixing

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