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**Duality in non-abelian algebra. IV. Duality for groups and a universal isomorphism theorem.**

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Summary: Abelian categories provide a self-dual axiomatic context for establishing homomorphism theorems (such as the isomorphism theorems and homological diagram lemmas) for abelian groups, and more generally, modules. In this paper we describe a self-dual context which allows one to establish the same theorems in the case of non-abelian group-like structures; the question of whether such a context can be found has been left open for seventy years. We also formulate and prove in our context a *universal isomorphism theorem* from which all other isomorphism theorems can be deduced.

**MSC:**

[20A05](#) Axiomatics and elementary properties of groups

[20J15](#) Category of groups

[16B50](#) Category-theoretic methods and results in associative algebras (except as in 16D90)

[08A30](#) Subalgebras, congruence relations

[06A15](#) Galois correspondences, closure operators (in relation to ordered sets)

[18D30](#) Fibered categories

[18E10](#) Abelian categories, Grothendieck categories

[18G50](#) Nonabelian homological algebra (category-theoretic aspects)

Cited in **3** Documents

**Keywords:**

[butterfly lemma](#); [connecting homomorphism](#); [duality for groups](#); [group-like structures](#); [isomorphism theorems](#); [semi-abelian category](#)

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