Tauber, C.; **Delplace, P.**; **Venaille, A.**

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Summary: Topology is introducing new tools for the study of fluid waves. The existence of unidirectional Yanai and Kelvin equatorial waves has been related to a topological invariant, the Chern number, that describes the winding of f-plane shallow water eigenmodes around band-crossing points in parameter space. In this previous study, the topological invariant was a property of the interface between two hemispheres. Here we ask whether a topological index can be assigned to each hemisphere. We show that this can be done if the shallow water model in the f-plane geometry is regularized by an additional odd-viscosity term. We then compute the spectrum of a shallow water model with a sharp equator separating two flat hemispheres, and recover the Kelvin and Yanai waves as two exponentially trapped waves along the equator, with all the other modes delocalized into the bulk. This model provides an exactly solvable example of bulk-interface correspondence in a flow with a sharp interface, and offers a topological interpretation for some of the transition modes described by *K. Iga* [ibid. 294, 367–390 (1995; Zbl 0842.76090)]. It also paves the way towards a topological interpretation of coastal Kelvin waves along a boundary and, more generally, to an understanding of bulk-boundary correspondence in continuous media.

MSC:

86A05 Hydrology, hydrography, oceanography
76U05 General theory of rotating fluids
76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

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