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Indistinguishability of conjugacy classes of the pro-l mapping class group. (English)

Zbl 0705.20037

Proc. Japan Acad., Ser. A 64, No. 7, 256-259 (1988).

From the introduction: "Let ℓ be a fixed prime number and $\pi^{(g)}$ denote the pro-ℓ completion of the topological fundamental group of a compact Riemann surface of genus $g \geq 2$. So, we have $\pi^{(g)} = F/N$, where F is the free pro-ℓ group of rank 2g generated by $x_1, ..., x_{2g}$ and N is the closed normal subgroup of F which is normally generated by $[x_1, x_{g+1}]...[x_g, x_{2g}]$, [,] being the commutator: $[x, y] = xyx^{-1}y^{-1}$ (x,y$\in$ F). We denote by Γ_g the outer automorphism group of $\pi^{(g)}$ and call it the pro-ℓ mapping class group. Let $\lambda : \Gamma_g \to GSp(2g, Z_\ell)$ be the canonical homomorphism induced by the action of Γ_g on $\pi^{(g)}/[\pi^{(g)}, \pi^{(g)}]$. We treat the case $g = 2$. Then, our result is the following Theorem: Assume that $\ell \geq 5$. Then, there exists an integer $N \geq 1$ such that the following statement holds: If $A \in GSp(4, Z_\ell)$ satisfies the condition $A \equiv \ell_4 mod \ell^N$, $\lambda^{-1}(C_A)$ contains more than one Γ_2-conjugacy class. Here, C_A denotes the $GSp(4, Z_\ell)$-conjugacy class containing A. In a previous paper, we have proved this "indistinguishability of conjugacy class" under the assumption that $g \geq 3$.... So, to prove the above theorem, we use the method "calculations modulo $\pi^{(g)}(4)''$. Although this requires rather complicated calculations, it is carried out by using the "Lie algebra" of the nilpotent pro-ℓ group $\pi^{(g)}/\pi^{(g)}(4)$."

Reviewer: T.Nôno

MSC:

20F34 Fundamental groups and their automorphisms (group-theoretic aspects)
20E18 Limits, profinite groups
30F10 Compact Riemann surfaces and uniformization
20F28 Automorphism groups of groups
20F40 Associated Lie structures for groups
20F14 Derived series, central series, and generalizations for groups

Keywords:

pro-ℓ completion; topological fundamental group; compact Riemann surface; free pro-ℓ group; outer automorphism group; pro-ℓ mapping class group; conjugacy class; Lie algebra; nilpotent pro-ℓ group

Full Text: DOI

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