Asada, Mamoru

An analogue of the Levi decomposition of the automorphism groups of certain nilpotent pro-ℓ groups. (English) Zbl 0705.20035

J. Algebra 132, No. 1, 160-169 (1990).

Let G be a finitely generated nilpotent pro-ℓ group, $\{G_k | \ k \in \mathbb{N}\}$ its descending central series with G_k/G_{k+1} being a free $\mathbb{Z}_\ell$-module of finite rank k. Denote by m the least integer with $G_m = (1)$. The author proves Theorem: for $\ell \geq m$ and the group Ω of bi-continuous automorphisms of G the short exact sequence $1 \to Ker\sigma \to \Omega^\sigma \to Aut(G/G_2) \to 1$ (with σ being the canonical homomorphism) splits. He notices also that there exists an automorphism $\sigma_\alpha \in \Omega$ such that $x_i^{\sigma_\alpha} = x_i^\alpha$ ($1\leq i \leq r$) for a given generating set $\{x_1, ..., x_r\}$ of G and $\alpha \in \mathbb{Z}_\ell^*$ satisfying $\alpha^j \neq 1$ ($1\leq j \leq m-2$). The author shows that the centralizer $C_\Omega(\sigma_\sigma)$ is independent of α and this subgroup $\Pi = C_\Omega(\sigma_\alpha)$ is such that $\Pi \cap Ker\sigma = (1)$ and $Im(\sigma|_\Pi) = Aut(G/G_2)$. The author notices also that Ω can be viewed as a linear ℓ-adic Lie group and $C(\sigma_\alpha)$ as its Levi subgroup. Two remarks are added: (1) for $m > \ell$ the above theorem isn't true in general, and (2) there exist hopes to give some application of the theorem to Galois representations.

Reviewer: U.Kaljulaid

MSC:

20F28 Automorphism groups of groups
20E18 Limits, profinite groups
20F14 Derived series, central series, and generalizations for groups
22E20 General properties and structure of other Lie groups

Keywords:

finitely generated nilpotent pro-ℓ group; descending central series; bi-continuous automorphisms; generating set; linear ℓ-adic Lie group; Levi subgroup

Full Text: DOI

References:

[1] Asada, M; Kaneko, M, On the automorphism group of some pro-\textit{l} fundamental groups, (), 137-159

[2] Bourbaki, N, Groupes et algebres de Lie, (1972), Hermann Paris, Chap. 2 et 3 · Zbl 0244.22007

[3] Deligne, P, A letter to S. Bloch, (February, 1984)

[4] Ihara, Y, Profinite braid groups, Galois representations and complex multiplications, Ann. of math., 123, 43-106, (1986) · Zbl 0595.12003

[5] Ihara, Y, On Galois representations arising from towers of coverings of \$P\$^{1}0,1,8, Invent. math., 86, 427-459, (1987)

[6] Ihara, Y, Some problems on three point ramifications and associated large Galois representations, (), 173-188

[7] Kaneko, M, (), [Japanese]

[8] Kaneko, M, On conjugacy classes of the pro-\textit{l} braid group of degree 2, (), 274-277 · Zbl 0618.20022

[9] Kohno, T; Oda, T, The lower central series of the pure braid group of an algebraic curve, (), 201-219

[10] Lazard, M, Sur LES groupes nilpotents et LES anneaux de Lie, Ann. sci. ecole norm. sup., 71, 101-190, (1954), (3) · Zbl 0055.25103

[11] Lazard, M, Groupes analytiques p-adiques, Inst. hautes etudes sci. publ. math., 26, (1965) · Zbl 0139.02302

[12] Oda, T, Two propositions on pro-\textit{l} braid groups, (1985), preprint

[13] Oda, T, Note on meta-abelian quotients of pro-\textit{l} free groups, (1985), preprint

[14] Weil, A, Sur LES groupes à \textit{p}n éléments, Rev. sci., 77, 321-322, (1939), (Collected papers, Vol. I, pp. 241-243) · Zbl 65.1126.01