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Rank/inertia approaches to weighted least-squares solutions of linear matrix equations. (English) Zbl 1426.15020

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Summary: The well-known linear matrix equation $AX = B$ is the simplest representative of all linear matrix equations. In this paper, we study quadratic properties of weighted least-squares solutions of this matrix equation. We first establish two groups of closed-form formulas for calculating the global maximum and minimum ranks and inertias of matrices in the two quadratical matrix-valued functions $Q_1 - XP_1X'$ and $Q_2 - X'P_2X$ subject to the restriction trace $[(AX - B)'W(AX - B)] = \min$, where both P_i and Q_i are real symmetric matrices, $i = 1, 2.W$ is a positive semi-definite matrix, and X' is the transpose of X. We then use the rank and inertia formulas to characterize quadratic properties of weighted least-squares solutions of $AX = B$, including necessary and sufficient conditions for weighted least-squares solutions of $AX = B$ to satisfy the quadratic symmetric matrix equalities $XP_1X' = Q_1$ an $X'P_2X = Q_2$, respectively, and necessary and sufficient conditions for the quadratic matrix inequalities $XP_1X' \succ Q_1 (\succcurlyeq Q_1, \prec Q_1, \preccurlyeq Q_1)$ and $X'P_2X \succ Q_2 (\succcurlyeq Q_2, \prec Q_2, \preccurlyeq Q_2)$ in the Löwner partial ordering to hold, respectively. In addition, we give closed-form solutions to four Löwner partial ordering optimization problems on $Q_1 - XP_1X'$ and $Q_2 - X'P_2X$ subject to weighted least-squares solutions of $AX = B$.

MSC:

15A24 Matrix equations and identities
15A09 Theory of matrix inversion and generalized inverses
62H12 Estimation in multivariate analysis
62J05 Linear regression; mixed models

Keywords:

matrix equation; weighted least-squares solution; quadratic matrix-valued function; rank; inertia; optimization

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