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Crystallographic bulk-edge correspondence: glide reflections and twisted mod 2 indices.

(English) [Zbl 1436.19011](#)

Lett. Math. Phys. 109, No. 4, 857-904 (2019).

The authors prove as the main result (Theorem 7.1 in this paper) that an index theorem for a twisted family of Toeplitz operators, as a generalization of the classical index theorem for Toeplitz operators, holds, in the sense that the topological push-forward (or bulk-edge) map of some \mathbb{Z}_2 -equivariant (or twisted) K -theory groups for $B_x \times B_y$ as a bundle over $B_x = \mathbb{R}/2\pi\mathbb{Z} = B_y$ as a circle S^1 with some sense, in the Gysin exact sequence in the topological K -theory is induced by the assignment to the input edge-bulk data U, V from the 1-dim Brillouin zone B_x and $B_x \times B_y$ both to the unitary group $U(2n)$, respectively, of the same mod 2 dimension as analytic invariant, of the kernels of the Toeplitz operators associated to the edge U as involving the multiplication operator for U on the Hilbert space $L^2(S^1)$.

There are many more details, omitted here in the review.

Reviewer: [Takahiro Sudo \(Nishihara\)](#)

MSC:

[19L50](#) Twisted K -theory; differential K -theory

[19L47](#) Equivariant K -theory

[19K56](#) Index theory

[47L80](#) Algebras of specific types of operators (Toeplitz, integral, pseudodifferential, etc.)

[47B35](#) Toeplitz operators, Hankel operators, Wiener-Hopf operators

[47A53](#) (Semi-) Fredholm operators; index theories

Cited in **2** Documents

Keywords:

bulk-edge correspondence; topological crystalline insulators; twisted K -theory; equivariant K -theory; Toeplitz index theorem; Toeplitz operator; Fredholm index; circle bundle

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