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Fast formal proof of the Erdős-Szekeres conjecture for convex polygons with at most 6 points. (English) Zbl 07038739

J. Autom. Reasoning 62, No. 3, 301-329 (2019).

Summary: A conjecture originally made by Klein and Szekeres in 1932 (now commonly known as "Erdős-Szekeres" or "Happy Ending" conjecture) claims that for every $m \geq 3$, every set of $2^{m-2}+1$ points in a general position (none three different points are collinear) contains a convex m-gon. The conjecture has been verified for $m \leq 6$. The case $m = 6$ was solved by Szekeres and Peters and required a huge computer enumeration that took "more than 3000 GHz hours". In this paper we improve the solution in several directions. By changing the problem representation, by employing symmetry-breaking and by using modern SAT solvers, we reduce the proving time to around only a half of an hour on an ordinary PC computer (i.e., our proof requires only around 1 GHz hour). Also, we formalize the proof within the Isabelle/HOL proof assistant, making it significantly more reliable.

MSC:

68T15 Theorem proving (deduction, resolution, etc.) (MSC2010)

Keywords:

Erdős-Szekeres conjecture; happy ending problem; convex polygons; interactive theorem proving; SAT solving; Isabelle/HOL

Software:

Coq; DRAT-trim; HOL Light; Isabelle/HOL

Full Text: DOI

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