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On Mahler's transcendence measure for e. (English) Zbl 1435.11095

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Given a transcendental number ξ one can define a transcendence measure $w(m,H)$ by the infinum of numbers $r>0$ such that

$$|\lambda_0+\lambda_1\xi+\cdots+\lambda_m\xi^m|>H^{-r}$$

for all integers λ_i with $|\lambda_i|\leq H$ and with $i=1,\dots,m$.

In the case of $\xi=e$ the authors provide explicit upper bounds for $w(m,H)$ provided that H is sufficiently large. These results improve earlier results due to *M. Hata* [J. Number Theory 54, No. 1, 81–92 (1995; Zbl 0839.11027)].

Reviewer: Volker Ziegler (Salzburg)

MSC:

11J82 Measures of irrationality and of transcendence
11J72 Irrationality; linear independence over a field
41A21 Padé approximation

Cited in **1** Document

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