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**G-space theory and weakened-weak form for micropolar media: application to smoothed point interpolation methods.** (English) Zbl 1464.74008  
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Summary: The concepts of G-space and weakened-weak form already available for the classic continuum model are extended to the case of the micropolar theory, in order to allow the use of smoothed point interpolation methods for the analysis of micropolar media. A new G-space, defined as a cartesian product of standard G-spaces, is introduced in order to represent both the displacement and the microrotation fields of a micropolar medium. Based on this G-space, a micropolar weakened-weak form is formulated, and the existence and uniqueness of its solution are proven.

**MSC:**

74A35 Polar materials

74S05 Finite element methods applied to problems in solid mechanics

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

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**Keywords:**

micropolar continuum theory; meshfree methods; smoothed point interpolation methods (S-PIMs); strain smoothing; weakened-weak form

**Software:**

Gmsh

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