Ducoffe, Guillaume

Finding cut-vertices in the square roots of a graph. (English) Zbl 1406.05101

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Summary: The square of a given graph $H = (V, E)$ is obtained from H by adding an edge between every two vertices at distance two in H. Given a graph class $\mathcal{H}$, the $\mathcal{H}$-square root problem asks for the recognition of the squares of graphs in $\mathcal{H}$. In this paper, we answer positively to an open question of *P. A. Golovach* et al. [Lect. Notes Comput. Sci. 9843, 361–372 (2016; Zbl 1391.68051)] by showing that the squares of cactus-block graphs can be recognized in polynomial time. Our proof is based on new relationships between the decomposition of a graph by cut-vertices and the decomposition of its square by clique cutsets. More precisely, we prove that the closed neighbourhoods of cut-vertices in H induce maximal subgraphs of $G = H^2$ with no clique-cutset. Furthermore, based on this relationship, we can compute from a given graph G the block-cut tree of a desired square root (if any). Although the latter tree is not uniquely defined, we show surprisingly that it can only differ marginally between two different roots. Our approach not only gives the first polynomial-time algorithm for the $\mathcal{H}$-square root problem for several graph classes $\mathcal{H}$, but it also provides a unifying framework for the recognition of the squares of trees, block graphs and cactus graphs – among others.

MSC:

05C85 Graph algorithms (graph-theoretic aspects)
68Q25 Analysis of algorithms and problem complexity
05C76 Graph operations (line graphs, products, etc.)

Keywords:

square root of a graph; clique-separator decomposition; cut-vertices; cactus-block graphs; cycle-power graphs

Software:

Algorithm 447

Full Text: DOI

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