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Tutte polynomials of two self-similar network models. (English) Zbl 1412.82011

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Summary: The Tutte polynomial $T(G;x,y)$ of a graph G, or equivalently the q-state Potts model partition function, is a two-variable polynomial graph invariant of considerable importance in combinatorics and statistical physics. Graph operations have been extensively applied to model complex networks recently. In this paper, we study the Tutte polynomials of the diamond hierarchical lattices and a class of self-similar fractal models which can be constructed through graph operations. Firstly, we find out the behavior of the Tutte polynomial under k-inflation and k-subdivision which are two graph operations. Secondly, we compute and gain the Tutte polynomials of this two self-similar fractal models by using their structure characteristic. Moreover, as an application of the obtained results, some evaluations of their Tutte polynomials are derived, such as the number of spanning trees and the number of spanning forests.

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
05C05 Trees
05C82 Small world graphs, complex networks (graph-theoretic aspects)
05C31 Graph polynomials

Keywords:

Tutte polynomial; number of spanning trees; complex network model; subdivision; inflation

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