Ernvall-Hytönen, Anne-Maria; Matala-Aho, Tapani; Seppälä, Louna
Euler's divergent series in arithmetic progressions. (English) Zbl 1443.11138
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The author considers the series $F(z) = \sum_{n=0}^{\infty} n!z^n$ which converges for $|z|_p \leq p^{1/(p-1)}$ in the p-adic numbers $\mathbb{Q}_p$. The corresponding function is denoted by $F_p(z)$ for a fixed prime p. From this perspective, it makes sense to ask whether $F_p(1)$ is irrational or not.

In the paper under review, the author proves that for a given rational number a/b there exist infinitely many primes p such that $F_p(1) \neq a/b$. Moreover, let $m \geq 3$. Then, the authors additionally show that infinitely many such primes p are contained in only $\varphi(m)/2$ residue classes modulo m.

Reviewer: Volker Ziegler (Salzburg)

MSC:

11J61 Approximation in non-Archimedean valuations

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