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A modified formulation of quasi-linear viscoelasticity for transversely isotropic materials under finite deformation. (English) Zbl 1407.74021

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Summary: The theory of quasi-linear viscoelasticity (QLV) is modified and developed for transversely isotropic (TI) materials under finite deformation. For the first time, distinct relaxation responses are incorporated into an integral formulation of nonlinear viscoelasticity, according to the physical mode of deformation. The theory is consistent with linear viscoelasticity in the small strain limit and makes use of relaxation functions that can be determined from small-strain experiments, given the time/deformation separability assumption. After considering the general constitutive form applicable to compressible materials, attention is restricted to incompressible media. This enables a compact form for the constitutive relation to be derived, which is used to illustrate the behaviour of the model under three key deformations: uniaxial extension, transverse shear and longitudinal shear. Finally, it is demonstrated that the Poynting effect is present in TI, neo-Hookean, modified QLV materials under transverse shear, in contrast to neo-Hookean elastic materials subjected to the same deformation. Its presence is explained by the anisotropic relaxation response of the medium.

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74D05 Linear constitutive equations for materials with memory

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