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Basin of attraction through invariant curves and dominant functions. (English)

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Summary: We study a second-order difference equation of the form $z_{n+1} = z_n F(z_{n-1}) + h$, where both $F(z)$ and $zF(z)$ are decreasing. We consider a set of invariant curves at $h = 1$ and use it to characterize the behaviour of solutions when $h > 1$ and when $0 < h < 1$. The case $h > 1$ is related to the Y2K problem. For $0 < h < 1$, we study the stability of the equilibrium solutions and find an invariant region where solutions are attracted to the stable equilibrium. In particular, for certain range of the parameters, a subset of the basin of attraction of the stable equilibrium is achieved by bounding positive solutions using the iteration of dominant functions with attracting equilibria.

MSC:

39A30 Stability theory for difference equations

39A20 Multiplicative and other generalized difference equations, e.g., of Lyness type

Full Text: [DOI](#)

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