Ernvall-Hytönen, Anne-Maria; **Odžak, Almasa**; **Sušić, Medina**

On asymptotic behavior of generalized Li coefficients. (English) Zbl 1429.11172

Taiwanese J. Math. 22, No. 6, 1321-1346 (2018).

Summary: In this paper, we consider the asymptotic behaviour of τ-Li coefficients for the wide class of L-functions that contains the Selberg class, the class of all automorphic L-functions, the Rankin-Selberg L-functions, as well as products of suitable shifts of the mentioned functions. We consider both archimedean and non-archimedean contribution to the τ-Li coefficients, both separately, and their joint contribution to the coefficients. We also derive the behavior of the coefficients in the case the $\tau/2$-Riemann hypothesis holds, which is the generalization of the Riemann hypothesis for the class under consideration. Finally, we conclude with some examples and numerics.

MSC:

11M41 Other Dirichlet series and zeta functions
11M26 Nonreal zeros of $\zeta(s)$ and $L(s,\chi)$; Riemann and other hypotheses

Keywords:

L-functions; generalized Li coefficients

Software:

Arb

Full Text: DOI Euclid

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