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Bifurcations of a two-dimensional discrete-time predator-prey model. (English) Zbl 07020831

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Summary: We study the local dynamics and bifurcations of a two-dimensional discrete-time predator-prey model in the closed first quadrant $\mathbb{R}_+^2$. It is proved that the model has two boundary equilibria: $O(0,0)$, $A(\frac{\alpha_1-1}{\alpha_1},0)$ and a unique positive equilibrium $B(\frac{1}{\alpha_2},\frac{\alpha_1\alpha_2-\alpha_1-\alpha_2}{\alpha_2})$ under some restriction to the parameter. We study the local dynamics along their topological types by imposing the method of linearization. It is proved that a fold bifurcation occurs about the boundary equilibria: $O(0,0)$, $A(\frac{\alpha_1-1}{\alpha_1},0)$ and a period-doubling bifurcation in a small neighborhood of the unique positive equilibrium $B(\frac{1}{\alpha_2},\frac{\alpha_1\alpha_2-\alpha_1-\alpha_2}{\alpha_2})$. It is also proved that the model undergoes a Neimark-Sacker bifurcation in a small neighborhood of the unique positive equilibrium $B(\frac{1}{\alpha_2},\frac{\alpha_1\alpha_2-\alpha_1-\alpha_2}{\alpha_2})$ and meanwhile a stable invariant closed curve appears. From the viewpoint of biology, the stable closed curve corresponds to the periodic or quasi-periodic oscillations between predator and prey populations. Numerical simulations are presented to verify not only the theoretical results but also to exhibit the complex dynamical behavior such as the period-2, -4, -11, -13, -15 and -22 orbits. Further, we compute the maximum Lyapunov exponents and the fractal dimension numerically to justify the chaotic behaviors of the discrete-time model. Finally, the feedback control method is applied to stabilize chaos existing in the discrete-time model.

MSC:

39A10 Additive difference equations
40A05 Convergence and divergence of series and sequences
92D25 Population dynamics (general)
70K50 Bifurcations and instability for nonlinear problems in mechanics
35B35 Stability in context of PDEs

Keywords:

discrete-time predator-prey model; stability and bifurcations; center manifold theorem; fractal dimension; chaos control; numerical simulation

Full Text: DOI

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