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About the domino problem for subshifts on groups. (English) Zbl 1405.20023

Berthé, Valérie (ed.) et al., Sequences, groups, and number theory. Cham: Birkhäuser (ISBN 978-3-319-69151-0/hbk; 978-3-319-69152-7/ebook). Trends in Mathematics, 331-389 (2018).

Summary: From a classical point of view, the domino problem is the question of the existence of an algorithm which can decide whether a finite set of square tiles with colored edges can tile the plane, subject to the restriction that adjacent tiles share the same color along their adjacent edges. This question has already been settled in the negative by *R. Berger* [Mem. Am. Math. Soc. 66, 72 p. (1966; [Zbl 0199.30802](#))]; however, these tilings can be reinterpreted in dynamical terms using the formalism of subshifts of finite type, and hence the same question can be formulated for arbitrary finitely generated groups. In this chapter, we present the state of the art concerning the domino problem in this extended framework. We also discuss different notions of effectiveness in subshifts defined over groups, that is, the ways in which these dynamical objects can be described through Turing machines.

For the entire collection see [[Zbl 1394.05002](#)].

MSC:

- [20F10](#) Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)
- [03D40](#) Word problems, etc. in computability and recursion theory
- [03D35](#) Undecidability and degrees of sets of sentences
- [05B45](#) Combinatorial aspects of tessellation and tiling problems

Full Text: [DOI](#)

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