Auletta, Vincenzo; **Ferraioli, Diodato**; **Pasquale, Francesco**; **Persiano, Giuseppe**
Metastability of logit dynamics for coordination games. (English) Zbl 1417.91092
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Summary: Logit dynamics (Blume in Games Econ Behav 5:387-424, 1993) are randomized best response dynamics for strategic games: at every time step a player is selected uniformly at random and she chooses a new strategy according to a probability distribution biased toward strategies promising higher payoffs. This process defines an ergodic Markov chain, over the set of strategy profiles of the game, whose unique stationary distribution is the long-term equilibrium concept for the game. However, when the mixing time of the chain is large (e.g., exponential in the number of players), the stationary distribution loses its appeal as equilibrium concept, and the transient phase of the Markov chain becomes important. It can happen that the chain is "metastable", i.e., on a time-scale shorter than the mixing time, it stays close to some probability distribution over the state space, while in a time-scale multiple of the mixing time it jumps from one distribution to another. In this paper we give a quantitative definition of "metastable probability distributions" for a Markov chain and we study the metastability of the logit dynamics for some classes of coordination games. We first consider a pure n-player coordination game that highlights the distinctive features of our metastability notion based on distributions. Then, we study coordination games on the clique without a risk-dominant strategy (which are equivalent to the well-known Glauber dynamics for the Curie-Weiss model) and coordination games on a ring (both with and without risk-dominant strategy).

MSC:

91A22 Evolutionary games
91A26 Rationality and learning in game theory
60J20 Applications of Markov chains and discrete-time Markov processes on general state spaces (social mobility, learning theory, industrial processes, etc.)

Full Text: DOI

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