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A new approach for solving a class of delay fractional partial differential equations. (English)

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Summary: In this article, a new numerical approach has been proposed for solving a class of delay time-fractional partial differential equations. The approximate solutions of these equations are considered as linear combinations of Müntz-Legendre polynomials with unknown coefficients. Operational matrix of fractional differentiation is provided to accelerate computations of the proposed method. Using Padé approximation and two-sided Laplace transformations, the mentioned delay fractional partial differential equations will be transformed to a sequence of fractional partial differential equations without delay. The localization process is based on the space-time collocation in some appropriate points to reduce the fractional partial differential equations into the associated system of algebraic equations which can be solved by some robust iterative solvers. Some numerical examples are also given to confirm the accuracy of the presented numerical scheme. Our results approved decisive preference of the Müntz-Legendre polynomials with respect to the Legendre polynomials.

MSC:

65M70	Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
35R11	Fractional partial differential equations
44A10	Laplace transform
42C10	Fourier series in special orthogonal functions (Legendre polynomials, Walsh functions, etc.)
65D32	Numerical quadrature and cubature formulas
33C45	Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords:

delay fractional partial differential equations; operational matrix; Müntz polynomials; pseudospectral method; Padé approximation; two-sided Laplace transformations

Software:

Maple

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