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Asymptotic distribution of values of isotropic here quadratic forms at S-integral points. (English) Zbl 1407.11084

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Summary: We prove an analogue of a theorem of Eskin-Margulis-Mozes [*A. Eskin* et al., Ann. Math. (2) 147, No. 1, 93–141 (1998; Zbl 0906.11035)]. Suppose we are given a finite set of places S over $\mathbb{Q}$ containing the Archimedean place and excluding the prime 2, an irrational isotropic form $\mathbf{q}$ of rank $n \geq 4$ on $\mathbb{Q}_S$, a product of p-adic intervals I_p, and a product Ω of star-shaped sets. We show that unless $n = 4$ and $\mathbf{q}$ is split in at least one place, the number of S -integral vectors $\mathbf{v} \in \mathsf{T}\Omega$ satisfying simultaneously $\mathbf{q}(\mathbf{v}) \in I_p$ for $p \in S$ is asymptotically given by

$$\lambda(\mathbf{q}, \Omega)|\mathsf{I}| \cdot \|\mathsf{T}\|^{n-2}$$

as T goes to infinity, where $|\mathsf{I}|$ is the product of Haar measures of the p-adic intervals I_p . The proof uses dynamics of unipotent flows on S-arithmetic homogeneous spaces; in particular, it relies on an equidistribution result for certain translates of orbits applied to test functions with a controlled growth at infinity, specified by an S-arithmetic variant of the α-function introduced in [loc. cit.], and an S-arithemtic version of a theorem of [*S. G. Dani* and *G. A. Margulis*, in: I. M. Gelfand Seminar. Part 1: Papers of the Gelfand seminar in functional analysis held at Moscow University, Russia, September 1993. Providence, RI: American Mathematical Society. 91–137 (1993; Zbl 0814.22003)].

MSC:

11H50 Minima of forms
22E40 Discrete subgroups of Lie groups
22D40 Ergodic theory on groups
11P21 Lattice points in specified regions
05C15 Coloring of graphs and hypergraphs
37E25 Dynamical systems involving maps of trees and graphs
68R15 Combinatorics on words

Keywords:

Oppenheim conjecture; homogeneous dynamic

Full Text: DOI

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