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New formulae for the decycling number of graphs. (English) Zbl 1401.05074
Discuss. Math., Graph Theory 39, No. 1, 125-141 (2019).

Summary: A set S of vertices of a graph G is called a decycling set if $G - S$ is acyclic. The minimum order of a decycling set is called the decycling number of G , and denoted by $\nabla(G)$. Our results include:

(a) For any graph G ,

$$\nabla(G) = n - \max_T \{\alpha(G - E(T))\},$$

where T is taken over all the spanning trees of G and $\alpha(G - E(T))$ is the independence number of the co-tree $G - E(T)$. This formula implies that computing the decycling number of a graph G is equivalent to finding a spanning tree in G such that its co-tree has the largest independence number. Applying the formula, the lower bounds for the decycling number of some (dense) graphs may be obtained. (b) For any decycling set S of a k -regular graph G ,

$$|S| = \frac{1}{k-1}(\beta(G) + m(S)),$$

where $\beta(G) = |E(G)| - |V(G)| + 1$ and $m(S) = c + |E(S)| - 1$, c and $|E(S)|$ are, respectively, the number of components of $G - S$ and the number of edges in $G[S]$. Hence S is a ∇ -set if and only if $m(S)$ is minimum, where ∇ -set denotes a decycling set containing exactly $\nabla(G)$ vertices of G . This provides a new way to locate $\nabla(G)$ for k -regular graphs G . (c) 4-regular graphs G with the decycling number $\nabla(G) \left\lceil \frac{\beta(G)}{3} \right\rceil$ are determined.

MSC:

- 05C07 Vertex degrees
- 05C38 Paths and cycles
- 05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)
- 05C42 Density (toughness, etc.)

Keywords:

decycling number; independence number; cycle rank; margin number

Full Text: [DOI](#)

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