Farano, Mirko; **Cherubini, Stefania**; **Robinet, J.-C.**; **De Palma, P.**; **Schneider, Tobias M.**
Computing heteroclinic orbits using adjoint-based methods. (English) Zbl 1415.76007
J. Fluid Mech. 858, R3, 13 p. (2019).

Summary: Transitional turbulence in shear flows is supported by a network of unstable exact invariant solutions of the Navier-Stokes equations. The network is interconnected by heteroclinic connections along which the turbulent trajectories evolve between invariant solutions. While many invariant solutions in the form of equilibria, travelling waves and periodic orbits have been identified, computing heteroclinic connections remains a challenge. We propose a variational method for computing orbits dynamically connecting small neighbourhoods around equilibrium solutions. Using local information on the dynamics linearized around these equilibria, we demonstrate that we can choose neighbourhoods such that the connecting orbits shadow heteroclinic connections. The proposed method allows one to approximate heteroclinic connections originating from states with multi-dimensional unstable manifold and thereby provides access to heteroclinic connections that cannot easily be identified using alternative shooting methods. For plane Couette flow, we demonstrate the method by recomputing three known connections and identifying six additional previously unknown orbits.

MSC:

76A02 Foundations of fluid mechanics
37N10 Dynamical systems in fluid mechanics, oceanography and meteorology
76M30 Variational methods applied to problems in fluid mechanics

Cited in **2** Documents

Keywords:

mathematical foundations; variational methods; nonlinear dynamical systems

Software:

channelflow

Full Text: DOI

References:

[1] Budanur, N. B.; Short, K. Y.; Farazmand, M.; Willis, A. P.; Cvitanović, P., Relative periodic orbits form the backbone of turbulent pipe flow, J. Fluid Mech., 833, 274-301, (2017)

[2] Chantry, M.; Schneider, T. M., Studying edge geometry in transiently turbulent shear flows, J. Fluid Mech., 747, 506-517, (2014)

[3] Cherubini, S.; De Palma, P., Minimal perturbations approaching the edge of chaos in a Couette flow, Fluid Dyn. Res., 46, 4, (2014)

[4] Cherubini, S.; De Palma, P.; Robinet, J.-C.; Bottaro, A., Rapid path to transition via nonlinear localized optimal perturbations in a boundary-layer flow, Phys. Rev. E, 82, 6, (2010) · Zbl 1197.76045

[5] Crommelin, D. T., Regime transitions and heteroclinic connections in a barotropic atmosphere, J. Atmos. Sci., 60, 2, 229-246, (2003)

[6] Dong, C.; Lan, Y., A variational approach to connecting orbits in nonlinear dynamical systems, Phys. Lett. A, 378, 9, 705-712, (2014) · Zbl 1331.37084

[7] Duguet, Y.; Willis, A. P.; Kerswell, R. R., Transition in pipe flow: the saddle structure on the boundary of turbulence, J. Fluid Mech., 613, 255-274, (2008) · Zbl 1151.76495

[8] Faisst, H.; Eckhardt, B., Travelling waves in pipe flow, Phys. Rev. Lett., 91, (2003)

[9] Farano, M.; Cherubini, S.; Robinet, J.-C.; De Palma, P., Hairpin-like optimal perturbations in plane Poiseuille flow, J. Fluid Mech., 775, R2, (2015)

[10] Farano, M.; Cherubini, S.; Robinet, J.-C. Robinet; De Palma, P.; Schneider, T. M.

[11] Foures, Dpg; Caulfield, Cp; Schmid, Pj, Localization of flow structures using \textit{⊠}-norm optimization, J. Fluid Mech., 729, 672-701, (2013) · Zbl 1291.76124

[12] Gibson, J. F.

[13] Gibson, J. F.; Halcrow, J.; Cvitanović, P., Visualizing the geometry of state space in plane Couette flow, J. Fluid Mech., 611, 107-130, (2008) · Zbl 1151.76453

[14] Gibson, J. F.; Halcrow, J.; Cvitanović, P., Equilibrium and traveling-wave solutions of plane Couette flow, J. Fluid Mech., 638, 243-266, (2009) · Zbl 1183.76688

[15] Halcrow, J.; Gibson, J. F.; Cvitanović, P.; Viswanath, D., Heteroclinic connections in plane Couette flow, J. Fluid Mech., 621, 365-376, (2009) · Zbl 1171.76383

[16] Hof, B.; Budanur, N. B., Heteroclinic path to spatially localized chaos in pipe flow, J. Fluid Mech., 827, R1, (2017)

[17] Hof, B.; Van Doorne, C. W. H.; Westerweel, J.; Nieuwstadt, F. T. M.; Faisst, H.; Eckhardt, B.; Wedin, H.; Kerswell, R. R.; Waleffe, F., Experimental observation of nonlinear traveling waves in turbulent pipe flow, Science, 305, 1594-1598, (2004)

[18] Hopf, E., A mathematical example displaying features of turbulence, Commun. Pure Appl. Maths, 1, 303-322, (1948) · Zbl 0031.32901

[19] Kawahara, G.; Kida, S., Periodic motion embedded in plane Couette turbulence: regeneration cycle and burst, J. Fluid Mech., 449, 291-300, (2001) · Zbl 0996.76034

[20] Kawahara, G.; Uhlmann, M.; Van Veen, L., The significance of simple invariant solutions in turbulent flows, Annu. Rev. Fluid Mech., 44, 203-225, (2012) · Zbl 1352.76031

[21] Kerswell, R. R., Nonlinear nonmodal stability theory, Annu. Rev. Fluid Mech., 50, 1, 319-345, (2018) · Zbl 1384.76022

[22] Krauskopf, B.; Osinga, H. M., Computing invariant manifolds via the continuation of orbit segments, Numerical Continuation Methods for Dynamical Systems, 117-154, (2007), Springer · Zbl 1126.65111

[23] Krauskopf, B.; Osinga, H. M.; Doedel, E. J.; Henderson, M. E.; Guckenheimer, J.; Vladimirsky, A.; Dellnitz, M.; Junge, O., A survey of methods for computing (un) stable manifolds of vector fields, Intl J. Bifurcation Chaos, 15, 3, 763-791, (2005) · Zbl 1086.34002

[24] Kreilos, T.; Veble, G.; Schneider, T. M.; Eckhardt, B., Edge states for the turbulence transition in the asymptotic suction boundary layer, J. Fluid Mech., 726, 100-122, (2013) · Zbl 1287.76122

[25] Lan, Y.; Cvitanović, P., Variational method for finding periodic orbits in a general flow, Phys. Rev. E, 69, 1, (2004)

[26] Nagata, M., Three-dimensional finite-amplitude solutions in plane Couette flow: bifurcation from infinity, J. Fluid Mech., 217, 519-527, (1990)

[27] Pringle, C. C. T.; Kerswell, R. R., Using nonlinear transient growth to construct the minimal seed for shear flow turbulence, Phys. Rev. Lett., 105, (2010)

[28] Schneider, T. M.; Gibson, J. F.; Lagha, M.; De Lillo, F.; Eckhardt, B., Laminar-turbulent boundary in plane Couette flow, Phys. Rev. E, 78, (2008)

[29] Suri, B.; Tithof, J.; Grigoriev, R. O.; Schatz, M. F., Forecasting fluid flows using the geometry of turbulence, Phys. Rev. Lett., 118, 11, (2017)

[30] Toh, S.; Itano, T., A periodic-like solution in channel flow, J. Fluid Mech., 481, 67-76, (2003) · Zbl 1034.76014

[31] Van Veen, L.; Kawahara, G., Homoclinic tangle on the edge of shear turbulence, Phys. Rev. Lett., 107, 11, (2011)

[32] Van Veen, L.; Kawahara, G.; Atsushi, M., On matrix-free computation of 2D unstable manifolds, SIAM J. Sci. Comput., 33, 1, 25-44, (2011) · Zbl 1227.65066

[33] Viswanath, D., Recurrent motions within plane Couette turbulence, J. Fluid Mech., 580, 339-358, (2007) · Zbl 1175.76074

[34] Waleffe, F., Homotopy of exact coherent structures in plane shear flows, Phys. Fluids, 15, 6, 1517-1534, (2003) · Zbl 1186.76556

[35] Wedin, H.; Kerswell, R. R., Exact coherent structures in pipe flow: traveling wave solutions, J. Fluid Mech., 508, 333-371, (2004) · Zbl 1065.76072

[36] Willis, A. P.; Cvitanović, P.; Avila, M., Revealing the state space of turbulent pipe flow by symmetry reduction, J. Fluid Mech., 721, 514-540, (2013) · Zbl 1287.76155