Krasko, Evgeniy; **Omelchenko, Alexander**

Enumeration of r-regular maps on the torus. I: Rooted maps on the torus, the projective plane and the Klein bottle. Sensed maps on the torus. (English) Zbl 1400.05019

Discrete Math. 342, No. 2, 584-599 (2019).

Summary: The work that consists of two parts is devoted to the problem of enumerating unrooted r-regular maps on the torus up to all its symmetries. We begin with enumerating near-r-regular rooted maps on the torus, the projective plane and the Klein bottle, as well as some special kinds of maps on the sphere: near-r-regular maps, maps with multiple leaves and maps with multiple root darts. For $r = 3$ and $r = 4$ we obtain exact analytical formulas. For larger r we derive recurrence relations. Then we enumerate r-regular maps on the torus up to homeomorphisms that preserve its orientation – so-called sensed maps. Using the concept of a quotient map on an orbifold we reduce this problem to enumeration of certain above-mentioned classes of rooted maps. For $r = 3$ and $r = 4$ we obtain closed-form expressions for the numbers of r-regular sensed maps by edges. All these results will be used in the second part of the work to enumerate r-regular maps on the torus up to all homeomorphisms – so-called unsensed maps.

Reviewer: Reviewer (Berlin)

MSC:

05A15 Exact enumeration problems, generating functions

Keywords:

map; surface; orbifold; unlabelled enumeration

Software:

OEIS

Full Text: DOI

References:

[1] Arques, D., Relations fonctionnelles et dénombrement des cartes pointées sur le tore, J. Combin. Theory Ser. B, 43, 3, 253-274, (1987) · Zbl 0628.05040

[2] Bender, E.; Canfield, E., The asymptotic number of rooted maps on a surface, J. Combin. Theory Ser. A, 43, 244-257, (1986) · Zbl 0606.05031

[3] Bender, E.; Canfield, E., The number of rooted maps on an orientable surface, J. Combin. Theory Ser. B, 53, 293-299, (1991) · Zbl 0751.05052

[4] Bender, E. A.; Canfield, E. R., The number of degree-restricted rooted maps on the sphere, SIAM J. Discrete Math., 7, 1, 9-15, (1994) · Zbl 0794.05048

[5] Bousquet-Mélou, M.; Jehanne, A., Polynomial equations with one catalytic variable, algebraic series and map enumeration, J. Comb. Theory Ser. B, 96, 5, 623-672, (2006) · Zbl 1099.05043

[6] Brown, W. G., Enumeration of triangulations of the disk, Proc. Lond. Math. Soc., 14, 3, 746-768, (1964) · Zbl 0134.19503

[7] Brown, W. G., Enumeration of quadrangular dissections of the disk, Canad. J. Math., 17, 302-317, (1965) · Zbl 0138.19104

[8] Brown, W. G., On the existence of square roots in certain rings of power series, Math. Ann., 158, 82-89, (1965) · Zbl 0136.02503

[9] Brown, W. G., On the enumeration of non-planar maps, Mem. Amer. Math. Soc., 65, 526-545, (1966) · Zbl 0115.40901

[10] Gao, Z., The number of rooted triangular maps on a surface, J. Combin. Theory Ser. B, 52, 236-249, (1991) · Zbl 0751.05053

[11] Gao, Z., The number of degree restricted maps on general surfaces, Discrete Math., 123, 47-63, (1993) · Zbl 0792.05072

[12] Gao, Z. C.; Liskovets, V. A.; Wormald, N., Enumeration of unrooted odd-valent regular planar maps, Ann. Combin., 13, 233-259, (2009) · Zbl 1229.05116

[13] Gao, Z. C.; Rahman, M., Enumeration of k-poles, Ann. Combin., 1, 55-66, (1997) · Zbl 0927.05038

[14] Gao, Z.; Wormald, N. C., Enumeration of rooted cubic planar maps, Ann. Comb., 6, 313-325, (2002) · Zbl 1017.05050

[15] Giorgetti, A.; Walsh, T. R.S., Efficient enumeration of rooted maps of a given orientable genus by number of faces and vertices, Ars Math. Contemp., 7, 2, 263-280, (2014) · Zbl 1317.05090

[16] Jackson, D. M.; Visentin, T. I., An Atlas of the Smaller Maps in Orientable and Non-Orientable Surfaces, (2000), Chapman and Hall

[17] Liskovets, V. A., Enumeration of nonisomorphic planar maps, Sel. Math. Sov., 4, 303-323, (1985) · Zbl 0578.05033

[18] Long, S.; Ren, H., Counting 2-connected 4-regular maps on the projective plane, Electron. J. Combin., 21, 2, (2014) · Zbl 1300.05131

[19] Mednykh, A.; Nedela, R., Enumeration of unrooted maps of a given genus, J. Combin. Theory Ser. B, 96, 5, 709-729, (2006) · Zbl 1102.05033

[20] Mullin, R. C., Enumeration of rooted triangular maps, Amer. Math. Monthly, 71, 1007-1010, (1964) · Zbl 0127.39204

[21] Mullin, R. C., On counting rooted triangular maps, Canad. J. Math., 17, 373-382, (1965) · Zbl 0142.41203

[22] Nedela, R.; d'Azevedo, A. B.; Mednykh, A., Enumeration of maps regardless of genus: Geometric approach, Discrete Math., 310, 1184-1203, (2010) · Zbl 1236.05109

[23] Nedela, R.; Mednykh, A., Enumeration of unrooted hypermaps of a given genus, Discrete Math., 310, 518-526, (2010) · Zbl 1185.05082

[24] OEIS Foundation Inc., The on-line encyclopedia of integer sequences, 2018. http://oeis.org.

[25] Omelchenko, A. V.; Krasko, E. S., Enumeration of 4-regular one-face maps, European J. Combin., 62, 167-177, (2017) · Zbl 1358.05015

[26] Ren, H.; Liu, Y., 4-regular maps on the Klein bottle, J. Combin. Theory Ser. B, 82, 118-137, (2001) · Zbl 1023.05077

[27] Ren, H.; Liu, Y., Enumeration of near-4-regular maps on the sphere and torus, Discrete Appl. Math., 110, 273-288, (2001) · Zbl 0979.05058

[28] Ren, H.; Liu, Y., The number of loopless 4-regular maps on the projective plane, J. Combin. Theory Ser. B, 84, 84-99, (2002) · Zbl 1018.05046

[29] Robinson, R. W.; Bender, E. A.; Canfield, E. R., The enumeration of maps on the torus and the projective plane, Canad. Math. Bull., 31, 257-271, (1988) · Zbl 0617.05036

[30] Singerman, D.; Jones, G. A., Theory of maps on orientable surfaces, Proc. Lond. Math. Soc., 37, 273-307, (1978) · Zbl 0391.05024

[31] Tutte, W. T., A census of planar triangulations, Canad. J. Math., 14, 21-38, (1962) · Zbl 0103.39603

[32] Tutte, W. T., A census of slicings, Canad. J. Math., 14, 708-722, (1962) · Zbl 0111.35202

[33] Tutte, W. T., A census of planar maps, Canad. J. Math., 15, 249-271, (1963) · Zbl 0115.17305

[34] Tutte, W. T., On the enumeration of planar maps, Bull. Amer. Math. Soc., 74, 64-74, (1968) · Zbl 0157.31101

[35] T.R.S. Walsh, A. Giorgetti, Constructing large tables of numbers of maps by orientable genus, 2014. https://arxiv.org/pdf/1405.0615.pdf.

[36] Walsh, T. R.S.; Lehman, A. B., Counting rooted maps by genus, I, J. Combin. Theory Ser. B, 13, 192-218, (1972) · Zbl 0228.05108

[37] West, D. B., Introduction to Graph Theory, (2002), Pearson Education, Inc.