Kreilos, Tobias; **Schneider, Tobias M.**

Fully localized post-buckling states of cylindrical shells under axial compression. (English)

Zbl 1402.74072

Proc. R. Soc. Lond., A, Math. Phys. Eng. Sci. 473, No. 2205, Article ID 20170177, 16 p. (2017).

Summary: We compute nonlinear force equilibrium solutions for a clamped thin cylindrical shell under axial compression. The equilibrium solutions are dynamically unstable and located on the stability boundary of the unbuckled state. A fully localized single dimple deformation is identified as the *edge state* – the attractor for the dynamics restricted to the stability boundary. Under variation of the axial load, the single dimple undergoes homoclinic snaking in the azimuthal direction, creating states with multiple dimples arranged around the central circumference. Once the circumference is completely filled with a ring of dimples, snaking in the axial direction leads to further growth of the dimple pattern. These fully nonlinear solutions embedded in the stability boundary of the unbuckled state constitute critical shape deformations. The solutions may thus be a step towards explaining when the buckling and subsequent collapse of an axially loaded cylinder shell is triggered.

MSC:

74K25 Shells
74G60 Bifurcation and buckling
74S30 Other numerical methods in solid mechanics (MSC2010)

Keywords:

cylinder buckling; shell buckling; edge state; homoclinic snaking

Software:

Eigen

Full Text: DOI

References:

[1] Hutchinson, JW, Knockdown factors for buckling of cylindrical and spherical shells subject to reduced biaxial membrane stress, Int. J. Solids Struct., 47, 1443-1448, (2010) · Zbl 1193.74045

[2] Koiter, WT, The stability of elastic equilibrium. Dissertation, Techische Hooge School Delft, The Netherlands, (1945)

[3] Arbocz, J., The effect of general imperfections on the buckling of cylindrical shells. PhD thesis, California Institute of Technology, Pasadena, CA, (1968)

[4] Thompson, JMT, Advances in shell buckling: theory and experiments, Int. J. Bifurcation Chaos, 25, 1530001, (2015)

[5] Thompson, JMT; Sieber, J., Shock-sensitivity in shell-like structures: with simulations of spherical shell buckling, Int. J. Bifurcation Chaos, 26, 1630003, (2016) · Zbl 1334.74038

[6] Lord, GJ; Champneys, AR; Hunt, GW, Computation of localized post buckling in long axially compressed cylindrical shells, Phil. Trans. R. Soc. Lond. A, 355, 2137-2150, (1997) · Zbl 0894.73040

[7] Lord, GJ; Champneys, AR; Hunt, GW, Computation of homoclinic orbits in partial differential equations: an application to cylindrical shell buckling, SIAM J. Sci. Comput., 21, 591-619, (1999) · Zbl 0954.74021

[8] Hunt, GW; Lord, GJ; Champneys, AR, Homoclinic and heteroclinic orbits underlying the post-buckling of axially-compressed cylindrical shells, Comput. Methods Appl. Mech. Eng., 170, 239-251, (1999) · Zbl 0958.74021

[9] Hunt, GW; Peletier, MA; Champneys, AR; Woods, PD; Wadee, MA; Budd, CJ; Lord, GJ, Cellular buckling in long structures, Nonlinear Dyn., 21, 3-29, (2000) · Zbl 0974.74024

[10] Hunt, G.; Lord, G.; Peletier, M., Cylindrical shell buckling: a characterization of localization and periodicity, Discret. Contin. Dyn. Syst., 3, 505-518, (2003) · Zbl 1046.74017

[11] Hunt, GW, Buckling in space and time, Nonlinear Dyn., 43, 29-46, (2006) · Zbl 1138.74332

[12] Horák, J.; Lord, GJ; Peletier, MA, Cylinder buckling: the mountain pass as an organizing center, SIAM J. Appl. Math., 66, 1793-1824, (2006) · Zbl 1134.35042

[13] Horák, J.; Lord, GJ; Peletier, MA, Numerical variational methods applied to cylinder buckling, SIAM J. Sci. Comput., 30,

1362-1386, (2008) · Zbl 1162.74050

[14] Choi, Y.; McKenna, P., A mountain pass method for the numerical solution of semilinear elliptic problems, Nonlinear Anal. Theory Methods Appl., 20, 417-437, (1993) · Zbl 0779.35032

[15] Weinan, E.; Ren, W.; Vanden-Eijnden, E., String method for the study of rare events, Phys. Rev. B, 66, 052301, (2002)

[16] Hutchinson, JW, Notes on Beams, Plates & Shells (lecture notes), (2012)

[17] Schaeffer, D.; Golubitsky, M., Boundary conditions and mode jumping in the buckling of a rectangular plate, Commun. Math. Phys., 69, 209-236, (1979) · Zbl 0414.73036

[18] Fornberg, B., Calculation of weights in finite difference formulas, SIAM Rev., 40, 685-691, (1998) · Zbl 0914.65010

[19] Ascher, UM; Ruuth, SJ; Wetton, BTR, Implicit-explicit methods for time-dependent partial differential equations, SIAM J. Numer. Anal., 32, 797-823, (1995) · Zbl 0841.65081

[20] Guennebaud, G.; Jacob, B., Eigen v3. http://eigen.tuxfamily.org, (2010)

[21] Skufca, JD; Yorke, JA; Eckhardt, B., Edge of chaos in a parallel shear flow, Phys. Rev. Lett., 96, 174101, (2006)

[22] Schneider, TM; Eckhardt, B., Edge of chaos in pipe flow, Chaos: Interdiscip. J. Nonlinear Sci., 16, 041103, (2006)

[23] Toh, S.; Itano, T., A periodic-like solution in channel flow, J. Fluid. Mech., 481, 67-76, (2003) · Zbl 1034.76014

[24] Schneider, TM, State space properties of transitional pipe flow. PhD thesis, Philipps-Universität Marburg, (2007)

[25] Gibson, JF, Channelflow: a spectral Navier-Stokes simulator in C++. Technical Report, University of New Hampshire, (2012)

[26] Dennis, JE; Schnabel, RB, $Numerical methods for unconstrained optimization and nonlinear equations$, (1996), SIAM

[27] Yamaki, N., $Elastic stability of circular cylindrical shells$, (1984), North-Holland · Zbl 0544.73062

[28] Knobloch, E., Spatial localization in dissipative systems, Ann. Rev. Condensed Mat. Phys., 6, 325-359, (2015)

[29] Swift, J.; Hohenberg, PC, Hydrodynamiic fluctuations at the convective instability, Phys. Rev. A, 15, 319-328, (1977)

[30] Batiste, O.; Knobloch, E.; Alonso, A.; Mercader, I., Spatially localized binary-fluid convection, J. Fluid Mech., 560, 149-158, (2006) · Zbl 1122.76029

[31] Schneider, TM; Gibson, JF; Burke, J., Snakes and ladders: localized solutions of plane Couette flow, Phys. Rev. Lett., 104, 104501, (2010)

[32] Gibson, JF; Schneider, TM, Homoclinic snaking in plane Couette flow: bending, skewing and finite-size effects, J. Fluid Mech., 794, 530-551, (2016)

[33] Haudin, F.; Rojas, RG; Bortolozzo, U.; Residori, S.; Clerc, MG, Homoclinic snaking of localized patterns in a spatially forced system, Phys. Rev. Lett., 107, 264101, (2011)

[34] Bergeon, A.; Burke, J.; Knobloch, E.; Mercader, I., Eckhaus instability and homoclinic snaking, Phys. Rev. E, 78, 046201, (2008)

[35] Lloyd, DJB; Sandstede, B.; Avitabile, D.; Champneys, AR, Localized hexagon patterns of the planar swift-Hohenberg equation, SIAM J. Appl. Dyn. Syst., 7, 1049-1100, (2008) · Zbl 1168.35311

[36] Beaume, C.; Bergeon, A.; Knobloch, E., Convectons and secondary snaking in three-dimensional natural doubly diffusive convection, Phys. Fluids, 25, 024105, (2013)