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Partitioning the edge set of a hypergraph into almost regular cycles. (English) Zbl 1402.05173

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Summary: A cycle of length t in a hypergraph is an alternating sequence $v_1, e_1, v_2, \dots, v_t, e_t$ of distinct vertices v_i and distinct edges e_i so that $\{v_i, v_{i+1}\} \subseteq e_i$(with $v_{t+1} := v_1$). Let λK_n^h be the λ-fold n-vertex complete h-graph. Let $\mathcal{G} = (V, E)$ be a hypergraph all of whose edges are of size at least h, and $2 \leq c_1 \leq \dots \leq c_k \leq |V|$. In order to partition the edge set of $\mathcal{G}$ into cycles of specified lengths $c_1, \dots, c_k$, an obvious necessary condition is that $\sum_{i=1}^{k} c_i = |E|$. We show that this condition is sufficient in the following cases.

(R1) $h \geq \max\{c_k, \lceil n/2 \rceil + 1\}$.

(R2) $\mathcal{G} = \lambda K_n^h,\ h \geq \lceil n/2 \rceil + 2$.

(R3) $\mathcal{G} = K_n^h,\ c_1 = \dots = c_k := c, c|n(n-1),\ n \geq 85$.

In (R2), we guarantee that each cycle is almost regular. In (R3), we also solve the case where a "small" subset L of edges of K_n^h is removed.

MSC:

05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
05C65 Hypergraphs
05C38 Paths and cycles
05C45 Eulerian and Hamiltonian graphs

Keywords:

almost regular; Baranyai's theorem; complete uniform hypergraph; cycle; circle; partition; Hamiltonicity; Kruskal-Katona theorem

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