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Sharpness of the phase transition for continuum percolation in $\mathbb{R}^2$. (English) Zbl 1404.60143

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Summary: We study the phase transition of random radii Poisson Boolean percolation: Around each point of a planar Poisson point process, we draw a disc of random radius, independently for each point. The behavior of this process is well understood when the radii are uniformly bounded from above. In this article, we investigate this process for unbounded (and possibly heavy tailed) radii distributions. Under mild assumptions on the radius distribution, we show that both the vacant and occupied sets undergo a phase transition at the same critical parameter λ_c. Moreover,

- For $\lambda < \lambda_c$, the vacant set has a unique unbounded connected component and we give precise bounds on the one-arm probability for the occupied set, depending on the radius distribution.
- At criticality, we establish the box-crossing property, implying that no unbounded component can be found, neither in the occupied nor the vacant sets. We provide a polynomial decay for the probability of the one-arm events, under sharp conditions on the distribution of the radius.
- For $\lambda > \lambda_c$, the occupied set has a unique unbounded component and we prove that the one-arm probability for the vacant decays exponentially fast.

The techniques we develop in this article can be applied to other models such as the Poisson Voronoi and confetti percolation.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
82B43 Percolation
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)

Keywords:

percolation; Poisson point processes; critical behavior; sharp thresholds

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