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Noise sensitivity and Voronoi percolation. (English) Zbl 1402.60122

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Summary: In this paper we study noise sensitivity and threshold phenomena for Poisson Voronoi percolation on $\mathbb{R}^2$. In the setting of Boolean functions, both threshold phenomena and noise sensitivity can be understood via the study of randomized algorithms. Together with a simple discretization argument, such techniques apply also to the continuum setting. Via the study of a suitable algorithm we show that box-crossing events in Voronoi percolation are noise sensitive and present a threshold phenomenon with polynomial window. We also study the effect of other kinds of perturbations, and emphasize the fact that the techniques we use apply for a broad range of models.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory

82B43 Percolation

60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)

Keywords:

noise sensitivity; Voronoi percolation; conservative perturbations

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