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Canonical extensions and ultraproducts of polarities. (English) Zbl 06963511
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Summary: Jónsson and Tarski's notion of the perfect extension of a Boolean algebra with operators has evolved into an extensive theory of canonical extensions of lattice-based algebras. After reviewing this evolution we make two contributions. First it is shown that the failure of a variety of algebras to be closed under canonical extensions is witnessed by a particular one of its free algebras. The size of the set of generators of this algebra can be made a function of a collection of varieties and is a kind of Hanf number for canonical closure. Secondly we study the complete lattice of stable subsets of a polarity structure, and show that if a class of polarities is closed under ultraproducts, then its stable set lattices generate a variety that is closed under canonical extensions. This generalises an earlier result of the author about generation of canonically closed varieties of Boolean algebras with operators, which was in turn an abstraction of the result that a first-order definable class of Kripke frames determines a modal logic that is valid in its so-called canonical frames.

MSC:

- [03G10](#) Logical aspects of lattices and related structures
- [06B23](#) Complete lattices, completions
- [03C20](#) Ultraproducts and related constructions
- [06A15](#) Galois correspondences, closure operators (in relation to ordered sets)
- [06D50](#) Lattices and duality

Cited in **5** Documents

Keywords:

[canonical extension](#); [canonical variety](#); [lattice](#); [completion](#); [lattice-based algebra](#); [MacNeille completion](#); [ultraproduct](#); [polarity](#); [Galois connection](#); [Hanf number](#)

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