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Basic principles and practical applications of the Cahn-Hilliard equation. (English)

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Summary: The celebrated Cahn-Hilliard (CH) equation was proposed to model the process of phase separation in binary alloys by Cahn and Hilliard. Since then the equation has been extended to a variety of chemical, physical, biological, and other engineering fields such as spinodal decomposition, diblock copolymer, image inpainting, multiphase fluid flows, microstructures with elastic inhomogeneity, tumor growth simulation, and topology optimization. Therefore, it is important to understand the basic mechanism of the CH equation in each modeling type. In this paper, we review the applications of the CH equation and describe the basic mechanism of each modeling type with helpful references and computational simulation results.

MSC:

[35K35](#) Initial-boundary value problems for higher-order parabolic equations
[35Q53](#) KdV equations (Korteweg-de Vries equations)
[74N20](#) Dynamics of phase boundaries in solids
[76T99](#) Multiphase and multicomponent flows
[80A22](#) Stefan problems, phase changes, etc.
[92B25](#) Biological rhythms and synchronization

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Full Text: [DOI](#)

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