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Dynamic trading under integer constraints. (English) Zbl 1416.91347

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Summary: In this paper, we investigate discrete-time trading under integer constraints, that is, we assume that the offered goods or shares are traded in integer quantities instead of the usual real quantity assumption. For finite probability spaces and rational asset prices, this has little effect on the core of the theory of no-arbitrage pricing. For price processes not restricted to the rational numbers, a novel theory of integer-arbitrage-free pricing and hedging emerges. We establish an FTAP, involving a set of absolutely continuous martingale measures satisfying an additional property. The set of prices of a contingent claim is not necessarily an interval, but is either empty or dense in an interval. We also discuss superhedging with integer-valued portfolios.

MSC:

91G10 Portfolio theory
60G44 Martingales with continuous parameter

Keywords:

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