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A constructive hypothesis test for the single-index models with two groups. (English)

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Authors' abstract: Comparison of two-sample heteroscedastic single-index models, where both the scale and location functions are modeled as single-index models, is studied in this paper. We propose a test for checking the equality of single-index parameters when dimensions of covariates of the two samples are equal. Further, we propose two test statistics based on Kolmogorov-Smirnov and Cramér-von Mises type functionals. These statistics evaluate the difference of the empirical residual processes to test the equality of mean functions of two single-index models. Asymptotic distributions of estimators and test statistics are derived. The Kolmogorov-Smirnov and Cramér-von Mises test statistics can detect local alternatives that converge to the null hypothesis at a parametric convergence rate. To calculate the critical values of Kolmogorov-Smirnov and Cramér-von Mises test statistics, a bootstrap procedure is proposed. Simulation studies and an empirical study demonstrate the performance of the proposed procedures.

Reviewer: [Fabio Rapallo \(Alessandria\)](#)

MSC:

- 62G08 Nonparametric regression and quantile regression
- 62G10 Nonparametric hypothesis testing
- 62G20 Asymptotic properties of nonparametric inference

Cited in **2** Documents

Keywords:

[empirical residual process](#); [single-index models](#); [local linear smoothing](#); [model checking](#)

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