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Maximum matching width: new characterizations and a fast algorithm for dominating set.
(English) Zbl 1395.05116
Discrete Appl. Math. 248, 114-124 (2018).

Summary: A graph of treewidth k has a representation by subtrees of a ternary tree, with subtrees of adjacent vertices sharing a tree node, and any tree node sharing at most $k+1$ subtrees. Likewise for branchwidth, but with a shift to the edges of the tree rather than the nodes. In this paper we show that the mm-width of a graph – maximum matching width – combines aspects of both these representations, targeting tree nodes for adjacency and tree edges for the parameter value. The proof of this new characterization of mm-width is based on a definition of canonical minimum vertex covers of bipartite graphs. We show that these behave in a monotone way along branch decompositions over the vertex set of a graph.

We use these representations to compare mm-width with treewidth and branchwidth, and also to give another new characterization of mm-width, by subgraphs of chordal graphs. We prove that given a graph G and a branch decomposition of maximum matching width k we can solve the Minimum Dominating Set Problem in time $O^*(8^k)$, thereby beating $O^*(3^{\mathrm{tw}(G)})$ whenever $\mathrm{tw}(G) > \log_3 8 \times k \approx 1.893k$. Note that $\mathrm{mmw}(G) \leq \mathrm{tw}(G) + 1 \leq 3\mathrm{mmw}(G)$ and these inequalities are tight. Given only the graph G and using the best known algorithms to find decompositions, maximum matching width will be better for minimum dominating set whenever $\mathrm{tw}(G) > 1.549 \times \mathrm{mmw}(G)$.

MSC:

05C62 Graph representations (geometric and intersection representations, etc.)
05C85 Graph algorithms (graph-theoretic aspects)
05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)
68Q25 Analysis of algorithms and problem complexity

Keywords:
treewidth; branchwidth; maximum matching width; minimum dominating set problem

Full Text: DOI

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