zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Żaba, Mariusz; **Garbaczewski, Piotr**; **Stephanovich, Vladimir**

Lévy flights in confining environments: random paths and their statistics. (English)

Zbl 1395.82089
Physica A 392, No. 17, 3485-3496 (2013).

Summary: We analyze a specific class of random systems that, while being driven by a symmetric Lévy stable noise, asymptotically set down at the Boltzmann-type equilibrium, represented by a probability density function (pdf) $\rho_*(x) \sim \exp[-\Phi(x)]$. This behavior needs to be contrasted with the standard Langevin representation of Lévy jump-type processes. It is known that the choice of the drift function in the Newtonian form $\sim -\nabla\Phi$ excludes the existence of the Boltzmannian pdf $\sim \exp[-\Phi(x)]$ (Eliazar-Klafter no go theorem). In view of this incompatibility statement, our main goal here is to establish the appropriate path-wise description of the equilibrating jump-type process. A priori given inputs are (i) jump transition rates entering the master equation for $\rho(x,t)$ and (ii) the target (invariant) pdf $\rho_*(x)$ of that equation, in the Boltzmannian form. We resort to numerical methods and construct a suitable modification of the Gillespie algorithm, originally invented in the chemical kinetics context. The generated sample trajectories show up a qualitative typicality, e.g. they display structural features of jumping paths (predominance of small vs large jumps) specific to particular stability indices $\mu \in (0,2)$. The obtained random paths statistical data allow us to infer an associated pdf $\rho(x,t)$ dynamics which stands for a validity check of our procedure. The considered exemplary Boltzmannian equilibria $\sim \exp[-\Phi(x)]$ refer to (i) harmonic potential $\Phi \sim x^2$, (ii) logarithmic potential $\Phi \sim n\ln(1+x^2)$ with $n=1,2$ and (iii) locally periodic confining potential $\Phi \sim \sin^2(2\pi x), |x| \leq 2$, $\Phi \sim (x^2-4), |x| > 2$.

MSC:

82B31 Stochastic methods applied to problems in equilibrium statistical mechanics
82B80 Numerical methods in equilibrium statistical mechanics (MSC2010)

Keywords:

symmetric stable noise; confining potentials; Boltzmann-type equilibrium; Gillespie's algorithm; random paths statistics; transport equations

Full Text: DOI

References:

[1] Risken, H., The Fokker-plack equation, (1989), Springer-Verlag Berlin
[2] Jespersen, S.; Metzler, R.; Fogedby, H. C., Phys. Rev. E, 59, 2736, (1999)
[3] Cohen, M. H.; Eliazar, I. I., Physica A, 392, 813, (2013)
[4] Metzler, R.; Klafter, J., J. Phys. A: Math. Gen., 37, R161, (2004)
[5] Brockmann, D.; Sokolov, I. M., Chem. Phys., 284, 409, (2002)
[6] Chen, L.; Deem, M. W., Phys. Rev. E, 65, 011109, (2001)
[7] Brockmann, D.; Geisel, T., Phys. Rev. Lett., 90, 170601, (2003)
[8] Belik, V. V.; Brockmann, D., New. J. Phys., 9, 54, (2007)
[9] Vilela Mendes, R., J. Math. Phys., 27, 178, (1986)
[10] Eleutério, S.; Vilela Mendes, R., Phys. Rev. B, 50, 5035, (1993)
[11] Garbaczewski, P.; Olkiewicz, R., J. Math. Phys., 40, 1057, (1999)
[12] Garbaczewski, P.; Olkiewicz, R., J. Math. Phys., 41, 6843, (2000)
[13] Garbaczewski, P.; Stephanovich, V. A., Phys. Rev. E, 80, 031113, (2009)
[14] Garbaczewski, P.; Stephanovich, V., Open Syst. Inf. Dyn., 17, 287, (2010)
[15] Garbaczewski, P.; Stephanovich, V. A., Physica A, 389, 4410, (2010)
[16] Garbaczewski, P.; Stephanovich, V., Phys. Rev. E, 84, 011142, (2011)

[17] Lőrinczi, J.; Hiroshima, F.; Betz, V., (Feyman-Kac-Type Theorems and Gibbs Measures on Path Space, Studies in Mathematics, vol. 34, (2011), Walter de Gruyter Berlin)

[18] Eliazar, I.; Klafter, J., J. Stat. Phys., 111, 739, (2003)

[19] Touchette, H.; Cohen, E. G.D., Phys. Rev. E, 76, 020101, (2007)

[20] Eliazar, I.; Klafter, J., J. Stat. Phys., 119, 165, (2005)

[21] Sato, K.-I., Lévy processes and infinitely divisible distributions, (1999), Cambridge University Press Cambridge · Zbl 0973.60001

[22] Janovsky, V. V., Physica A, 282, 13, (2000)

[23] Dubkov, A.; Spagnolo, B., Fluct. Noise Lett., 5, L267, (2005)

[24] Mantegna, R. N.; Stanley, H. E., Phys. Rev. Lett., 73, 2946, (1994)

[25] I.M. Sokolov, private communication.

[26] Gillespie, T. D., J. Phys. Chem., 8, 25, 2340, (1977)

[27] Gillespie, T. D., J. Comput. Phys., 22, 4, 403, (1976)

[28] C-code is available upon request.

[29] Matsumoto, M.; Nishimura, T., ACM Trans. Model. Comput. Simul., 8, 3, (1998)

[30] Janicki, A.; Weron, A., Simulation and chaotic behavior of α-stable stochastic processes, (1994), M. Dekker New York

[31] Hänggi, P.; Marchesoni, F., Rev. Mod. Phys., 81, 387, (2009)

[32] Denisov, S. I.; Lyutyy, T. V., Phys. Rev., 79, 051102, (2009)

[33] Gimperlein, H.; Wessel, S.; Schmiedmayer, J.; Santos, L., Phys. Rev. Lett., 95, 170401, (2005)

[34] Snider, J., Phys. Rev. E, 83, 011105, (2011)