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Developments of Mindlin-Reissner plate elements. (English) Zbl 1394.74099
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Summary: Since 1960s, how to develop high-performance plate bending finite elements based on different plate theories has attracted a great deal of attention from finite element researchers, and numerous models have been successfully constructed. Among these elements, the most popular models are usually formulated by two theoretical bases: the Kirchhoff plate theory and the Mindlin-Reissener plate theory. Due to the advantages that only C^0 continuity is required and the effect of transverse shear strain can be included, the latter one seems more rational and has obtained more attention. Through abundant works, different types of Mindlin-Reissener plate models emerged in many literatures and have been applied to solve various engineering problems. However, it also brings FEM users a puzzle of how to choose a "right" one. The main purpose of this paper is to present an overview of the development history of the Mindlin-Reissner plate elements, exhibiting the state-of-art in this research field. At the end of the paper, a promising method for developing "shape-free" plate elements is recommended.

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