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Global regularity for the 2D micropolar equations with fractional dissipation. (English)

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Summary: Micropolar equations, modeling micropolar fluid flows, consist of coupled equations obeyed by the evolution of the velocity u and that of the microrotation w. This paper focuses on the two-dimensional micropolar equations with the fractional dissipation $(-\Delta)^\alpha u$ and $(-\Delta)^\beta w$, where $0<\alpha,\beta<1$. The goal here is the global regularity of the fractional micropolar equations with minimal fractional dissipation. Recent efforts have resolved the two borderline cases $\alpha=1$, $\beta=0$ and $\alpha=0$, $\beta=1$. However, the situation for the general critical case $\alpha+\beta=1$ with $0<\alpha<1$ is far more complex and the global regularity appears to be out of reach. When the dissipation is split among the equations, the dissipation is no longer as efficient as in the borderline cases and different ranges of α and β require different estimates and tools. We aim at the subcritical case $\alpha+\beta>1$ and divide $\alpha\in(0,1)$ into five sub-intervals to seek the best estimates so that we can impose the minimal requirements on α and β. The proof of the global regularity relies on the introduction of combined quantities, sharp lower bounds for the fractional dissipation and delicate upper bounds for the nonlinearity and associated commutators.

MSC:

35Q35 PDEs in connection with fluid mechanics
35B65 Smoothness and regularity of solutions to PDEs
76A10 Viscoelastic fluids
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids
35B45 A priori estimates in context of PDEs
35R11 Fractional partial differential equations
76U05 General theory of rotating fluids

Keywords:

micropolar equation; fractional dissipation; global regularity

Full Text: DOI

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