Palhazi Cuervo, Daniel; **Goos, Peter**; **Sörensen, Kenneth**

An algorithmic framework for generating optimal two-stratum experimental designs. (English) Zbl 06917786

Comput. Stat. Data Anal. 115, 224-249 (2017).

Summary: Two-stratum experiments are widely used in the event a complete randomization is not possible. In some experimental scenarios, there are constraints that limit the number of observations that can be made under homogeneous conditions. In other scenarios, there are factors whose levels are hard or expensive to change. In both of these scenarios, it is necessary to arrange the observations in different groups. Moreover, it is important that the analysis performed accounts for the variation in the response variable due to the differences between the groups. The most common strategy for the design of these kinds of experiments is to consider groups of equal size. The number of groups and the number of observations per group are usually defined by the constraints that limit the experimental scenario. It is argued, however, that these constraints do not define the design itself, but should be considered only as upper bounds. The number of groups and the number of observations per group should be chosen not only to satisfy the experimental constraints, but also to maximize the quality of the experiment. An algorithmic framework for generating optimal designs for two-stratum experiments, in which the number of groups and the number of observations per group are limited only by upper bounds, is proposed. Computational results show that this additional flexibility in the design generation process can significantly improve the quality of the experiments. Additionally, the results also show that the grouping configuration of an optimal design depends on the characteristics of the two-stratum experiment, namely, the type of experiment, the model to be estimated and the optimality criterion considered. This is a strong argument in favor of using algorithmic techniques that are able to identify not only the best factor-level configuration for each experimental run, but also the best grouping configuration.

MSC:

62 Statistics

Keywords:

blocked experiments; split-plot experiments; two-stratum experiments; coordinate-exchange algorithm; variable neighborhood search

Full Text: DOI

References:

[1] Anbari, F.T., Lucas, J.M., 1994. Super-efficient designs: how to run your experiment for higher efficiency and lower cost. In: Annual Quality Congress Proceedings-American Society for Quality Control, pp. 853-853

[2] Anbari, F. T.; Lucas, J. M., Designing and running super-efficient experiments: optimum blocking with one hard-to-change factor, J. Qual. Technol., 40, 1, 31-45, (2008)

[3] Anderson, V. L.; McLean, R. A., Design of Experiments: A Realistic Approach, Vol. 5, (1974), CRC Press · Zbl 0277.62060

[4] Arnouts, H.; Goos, P., Staggered-level designs for experiments with more than one hard-to-change factor, Technometrics, 54, 4, 355-366, (2012)

[5] Arnouts, H.; Goos, P., Staggered-level designs for response surface modeling, J. Qual. Technol., 47, 2, 156-175, (2015)

[6] Arnouts, H.; Goos, P.; Jones, B., Design and analysis of industrial strip-plot experiments, Qual. Reliab. Eng. Int., 26, 2, 127-136, (2010)

[7] Atkinson, A. C.; Donev, A. N., The construction of exact D-optimum experimental designs with application to blocking response surface designs, Biometrika, 76, 3, 515-526, (1989) · Zbl 0677.62066

[8] Atkinson, A. C.; Donev, A. N.; Tobias, R., Optimum Experimental Designs, with SAS, Vol. 34, (2007), Oxford University Press USA · Zbl 1183.62129

[9] Bingham, D.; Schoen, E. D.; Sitter, R. R., Designing fractional factorial split-plot experiments with few whole-plot factors, J. R. Stat. Soc. Ser. C. Appl. Stat., 53, 2, 325-339, (2004) · Zbl 1111.62316

[10] Bingham, D.; Schoen, E. D.; Sitter, R. R., Corrigendum: designing fractional factorial split-plot experiments with few whole-

plot factors, J. R. Stat. Soc. Ser. C. Appl. Stat., 54, 5, 955-958, (2005)

[11] Bingham, D.; Sitter, R. R., Minimum-aberration two-level fractional factorial split-plot designs, Technometrics, 41, 1, 62-70, (1999)

[12] Bisgaard, S., Blocking generators for small $2^{k - p}$ designs, J. Qual. Technol., 26, 4, 288-296, (1994)

[13] Borrotti, M.; Sambo, F.; Mylona, K.; Gilmour, S., A multi-objective coordinate-exchange two-phase local search algorithm for multi-stratum experiments, Stat. Comput., 27, 2, 469-481, (2017) · Zbl 06697668

[14] Butler, N. A., Construction of two-level split-plot fractional factorial designs for multistage processes, Technometrics, 46, 4, 445-451, (2004)

[15] Cao, Y.; Smucker, B. J.; Robinson, T. J., On using the hypervolume indicator to compare Pareto fronts: applications to multi-criteria optimal experimental design, J. Statist. Plann. Inference, 160, 60-74, (2015) · Zbl 1311.62115

[16] Cao, Y.; Smucker, B. J.; Robinson, T. J., A hybrid elitist Pareto-based coordinate exchange algorithm for constructing multi-criteria optimal experimental designs, Stat. Comput., 27, 2, 423-437, (2017) · Zbl 06697665

[17] Cao, Y.; Wulff, S. S.; Robinson, T. J., DP-optimality in terms of multiple criteria and its application to the split-plot design, J. Qual. Technol., 49, 1, 27-45, (2017)

[18] Chasalow, S.D., 1992. Exact Response Surface Designs with Random Block Effects (Ph.D. thesis), University of California, Berkeley

[19] Chen, B.-J.; Li, P.-F.; Liu, M.-Q.; Zhang, R.-C., Some results on blocked regular 2-level fractional factorial designs with clear effects, J. Statist. Plann. Inference, 136, 12, 4436-4449, (2006) · Zbl 1099.62082

[20] Cheng, S. W.; Wu, C. F.J., Choice of optimal blocking schemes in two-level and three-level designs, Technometrics, 44, 3, 269-277, (2002)

[21] Cook, R. D.; Nachtsheim, C. J., Computer-aided blocking of factorial and response-surface designs, Technometrics, 31, 3, 339-346, (1989) · Zbl 0705.62072

[22] Fedorov, V. V., Theory of Optimal Experiments, (1972), Academic Press New York

[23] Ganju, J., On choosing between fixed and random block effects in some no-interaction models, J. Statist. Plann. Inference, 90, 2, 323-334, (2000) · Zbl 0958.62069

[24] Gilmour, S. G.; Goos, P., Analysis of data from non-orthogonal multistratum designs in industrial experiments, J. R. Stat. Soc. Ser. C. Appl. Stat., 58, 4, 467-484, (2009)

[25] Gilmour, S. G.; Trinca, L. A., Some practical advice on polynomial regression analysis from blocked response surface designs, Commun. Stat. - Theory Methods, 29, 9-10, 2157-2180, (2000) · Zbl 1061.62550

[26] Goos, P., The Optimal Design of Blocked and Split-Plot Experiments, (2002), Springer New York · Zbl 1008.62068

[27] Goos, P., Optimal versus orthogonal and equivalent-estimation design of blocked and split-plot experiments, Stat. Neerl., 60, 3, 361-378, (2006) · Zbl 1108.62073

[28] Goos, P.; Jones, B., Optimal Design of Experiments: A Case Study Approach, (2011), Wiley

[29] Goos, P.; Mylona, K., Quadrature methods for Bayesian optimal design of experiments with non-normal prior distributions, J. Comput. Graph. Statist., (2017), (in press)

[30] Goos, P.; Vandebroek, M., D-optimal response surface designs in the presence of random block effects, Comput. Statist. Data Anal., 37, 4, 433-453, (2001) · Zbl 1079.62532

[31] Goos, P.; Vandebroek, M., Optimal split-plot designs, J. Qual. Technol., 33, 4, 436-450, (2001)

[32] Goos, P.; Vandebroek, M., D-optimal split-plot designs with given numbers and sizes of whole plots, Technometrics, 45, 3, 235-245, (2003)

[33] Goos, P.; Vandebroek, M., Outperforming completely randomized designs, J. Qual. Technol., 36, 1, 12-26, (2004)

[34] Hansen, P.; Mladenović, N., Variable neighborhood search, (Glover, F.; Kochenberger, G. A., Handbook of Metaheuristics, International Series in Operations Research & Management Science, vol. 57, (2003), Springer), 145-184 · Zbl 1102.90371

[35] Hardin, R.H., Sloane, N.J.A., 1991. Computer-generated minimal (and larger) response-surface designs: (II) the cube, Tech. rep., Mathematical Sciences Research Center - AT&T Bell Laboratories

[36] Huang, P.; Chen, D.; Voelkel, J. O., Minimum-aberration two-level split-plot designs, Technometrics, 40, 4, 314-326, (1998) · Zbl 1064.62552

[37] Jones, B.; Goos, P., A candidate-set-free algorithm for generating D-optimal split-plot designs, J. R. Stat. Soc. Ser. C. Appl. Stat., 56, 3, 347-364, (2007)

[38] Jones, B.; Goos, P., D-optimal design of split-split-plot experiments, Biometrika, 96, 1, 67-82, (2009) · Zbl 1162.62396

[39] Jones, B.; Nachtsheim, C. J., Split-plot designs: what, why, and how, J. Qual. Technol., 41, 4, 340-361, (2009)

[40] Kessels, R.; Goos, P.; Vandebroek, M., Optimal designs for conjoint experiments, Comput. Statist. Data Anal., 52, 5, 2369-2387, (2008) · Zbl 1452.62581

[41] Khuri, A. I., Response surface models with random block effects, Technometrics, 34, 1, 26-37, (1992) · Zbl 0850.62618

[42] Letsinger, J. D.; Myers, R. H.; Lentner, M., Response surface methods for bi-randomization structures, J. Qual. Technol., 28, 4, 381-397, (1996)

[43] Mee, R. W.; Bates, R. L., Split-plot designs: experiments for multistage batch processes, Technometrics, 40, 2, 127-140, (1998)

[44] Meyer, R. K.; Nachtsheim, C. J., The coordinate-exchange algorithm for constructing exact optimal experimental designs,

Technometrics, 37, 1, 60-69, (1995) · Zbl 0825.62652

[45] Miller, A., Strip-plot configurations of fractional factorials, Technometrics, 39, 2, 153-161, (1997) · Zbl 0889.62069

[46] Mladenović, N.; Hansen, P., Variable neighborhood search, Comput. Oper. Res., 24, 11, 1097-1100, (1997) · Zbl 0889.90119

[47] Mylona, K.; Goos, P.; Jones, B., Optimal design of blocked and split-plot experiments for fixed effects and variance component estimation, Technometrics, 56, 2, 132-144, (2014)

[48] Palhazi Cuervo, D.; Goos, P.; Sörensen, K., Optimal design of large-scale screening experiments: a critical look at the coordinate-exchange algorithm, Stat. Comput., 26, 1, 15-28, (2016) · Zbl 1342.62132

[49] Sambo, F.; Borrotti, M.; Mylona, K., A coordinate-exchange two-phase local search algorithm for the D- and I-optimal designs of split-plot experiments, Comput. Statist. Data Anal., 71, 1193-1207, (2014)

[50] Sitter, R. R.; Chen, J.; Feder, M., Fractional resolution and minimum aberration in blocked $2^{n - k}$ designs, Technometrics, 39, 4, 382-390, (1997) · Zbl 0913.62073

[51] Smucker, B. J.; del Castillo, E.; Rosenberger, J. L., Model-robust designs for split-plot experiments, Comput. Statist. Data Anal., 56, 12, 4111-4121, (2012) · Zbl 1255.62222

[52] Sun, D. X.; Wu, C. F.J.; Chen, Y., Optimal blocking schemes for 2^n and $2^{n - p}$ designs, Technometrics, 39, 3, 298-307, (1997) · Zbl 0891.62055

[53] Trinca, L. A.; Gilmour, S. G., Difference variance dispersion graphs for comparing response surface designs with applications in food technology, J. R. Stat. Soc. Ser. C. Appl. Stat., 48, 4, 441-455, (1999) · Zbl 0956.62061

[54] Trinca, L. A.; Gilmour, S. G., An algorithm for arranging response surface designs in small blocks, Comput. Statist. Data Anal., 33, 1, 25-43, (2000) · Zbl 1061.62551

[55] Trinca, L. A.; Gilmour, S. G., An algorithm for arranging response surface designs in small blocks (erratum), Comput. Statist. Data Anal., 40, 3, 475, (2002)

[56] Trinca, L. A.; Gilmour, S. G., Multistratum response surface designs, Technometrics, 43, 1, 25-33, (2001) · Zbl 1072.62623

[57] Trinca, L. A.; Gilmour, S. G., Improved split-plot and multi-stratum designs, Technometrics, 57, 2, 145-154, (2015)

[58] Wang, J.; Yuan, Y.; Zhao, S., Fractional factorial split-plot designs with two-and four-level factors containing clear effects, Commun. Stat. - Theory Methods, 44, 4, 671-682, (2015) · Zbl 1317.62063

[59] Wu, C. F.J.; Chen, Y., A graph-aided method for planning two-level experiments when certain interactions are important, Technometrics, 34, 2, 162-175, (1992)

[60] Zhang, R.; Park, D., Optimal blocking of two-level fractional factorial designs, J. Statist. Plann. Inference, 91, 1, 107-121, (2000) · Zbl 0958.62072

[61] Zhao, S.; Chen, X., Mixed two-and four-level fractional factorial split-plot designs with clear effects, J. Statist. Plann. Inference, 142, 7, 1789-1793, (2012) · Zbl 1238.62089