

**Chen, Dawei; Möller, Martin; Zagier, Don**

**Quasimodularity and large genus limits of Siegel-Veech constants.** (English) Zbl 1404.32025  
*J. Am. Math. Soc.* 31, No. 4, 1059-1163 (2018).

The study of the Masur-Veech volumes of moduli spaces of translation surfaces and related counting problems on translation surfaces has connections to number theory, representation theory, dynamics, and geometry. The paper under review continues and adds to a part of this story that starts with work of Eskin-Okounkov, who related computing volumes to counting lattice points (*square-tiled surfaces*) in these moduli spaces, and showed that a related generating function was a quasi-modular form, and used this to compute some of these volumes. This deep study discovers new and beautiful combinatorial and representation theoretic properties of *Bloch-Okounkov correlators* and associated growth polynomials and uses these to give a proof of the Eskin-Zorich conjecture on large-genus asymptotics of the volume of the principal stratum of translation surfaces (this conjecture has been subsequently verified in all cases by Agarwal).

Reviewer: [Jayadev Athreya \(Seattle\)](#)

**MSC:**

- [32G15](#) Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
- [05A17](#) Combinatorial aspects of partitions of integers
- [11F23](#) Relations with algebraic geometry and topology
- [37A25](#) Ergodicity, mixing, rates of mixing
- [57M12](#) Low-dimensional topology of special (e.g., branched) coverings

Cited in **2** Reviews  
Cited in **9** Documents

**Keywords:**

[quasi-modular forms](#); [translation surfaces](#); [large-genus limits](#)

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