Duminil-Copin, H.; **Tassion, Vincent**; **Teixeira, Augusto**

The box-crossing property for critical two-dimensional oriented percolation. (English)

Zbl 1428.60138
Probab. Theory Relat. Fields 171, No. 3-4, 685-708 (2018).

Summary: We consider critical oriented Bernoulli percolation on the square lattice $\mathbb{Z}^2$. We prove a Russo-Seymour-Welsh type result which allows us to derive several new results concerning the critical behavior:

- We establish that the probability that the origin is connected to distance n decays polynomially fast in n.
- We prove that the critical cluster of 0 conditioned to survive to distance n has a typical width w_n satisfying $\varepsilon n^{2/5} \leq w_n \leq n^{1-\varepsilon}$ for some $\varepsilon > 0$.

The sub-linear polynomial fluctuations contrast with the supercritical regime where w_n is known to behave linearly in n. It is also different from the critical picture obtained for non-oriented Bernoulli percolation, in which the scaling limit is non-degenerate in both directions. All our results extend to the graphical representation of the one-dimensional contact process.

MSC:

60K35	Interacting random processes; statistical mechanics type models; percolation theory
82B43	Percolation
82C43	Time-dependent percolation in statistical mechanics

Keywords:

percolation; oriented percolation; critical behaviour; contact process; renormalization

Full Text: DOI

References:

[1] Balister, P; Bollobás, B; Stacey, A, Improved upper bounds for the critical probability of oriented percolation in two dimensions, Random Struct. Algorithms, 5, 573-589, (1994) · Zbl 0807.60092

[2] Bezuidenhout, C; Grimmett, G, The critical contact process dies out, Ann. Probab., 18, 1462-1482, (1990) · Zbl 0718.60109

[3] Broadbent, SR; Hammersley, JM, Percolation processes. I. crystals and mazes, Proc. Camb. Philos. Soc., 53, 629-641, (1957) · Zbl 0091.13901

[4] Belitsky, V; Ritchie, TL, Improved lower bounds for the critical probability of oriented bond percolation in two dimensions, J. Stat. Phys., 122, 279-302, (2006) · Zbl 1127.82025

[5] Duminil-Copin, H; Tassion, V, A new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model, Comm. Math. Phys., 343, 725-745, (2016) · Zbl 1342.82026

[6] Duminil-Copin, H., Tassion, V.: Rsw and box-crossing property for planar percolation. In: Proceedings of the International Congress of Mathematical Physics (2016) · Zbl 1342.82026

[7] Durrett, R; Griffeath, D, Supercritical contact processes on ${ Z}$, Ann. Probab., 11, 1-15, (1983) · Zbl 0508.60080

[8] Durrett, R; Schonmann, RH; Tanaka, NI, The contact process on a finite set. III. the critical case, Ann. Probab., 17, 1303-1321, (1989) · Zbl 0692.60085

[9] Durrett, R; Schonmann, RH; Tanaka, NI, Correlation lengths for oriented percolation, J. Stat. Phys., 55, 965-979, (1989) · Zbl 0714.60097

[10] Durrett, R; Tanaka, NI, Scaling inequalities for oriented percolation, J. Stat. Phys., 55, 981-995, (1989) · Zbl 0714.60098

[11] Durrett, R, Oriented percolation in two dimensions, Ann. Probab., 12, 999-1040, (1984) · Zbl 0567.60095

[12] Galves, A; Presutti, E, Edge fluctuations for the one-dimensional supercritical contact process, Ann. Probab., 15, 1131-1145, (1987) · Zbl 0645.60103

[13] Griffeath, D, The basic contact processes, Stoch. Process. Appl., 11, 151-185, (1981) · Zbl 0463.60085

[14] Grimmett, G.: Percolation, 2nd ed., Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 321. Springer, Berlin (1999) · Zbl 1127.82025

[15] Harris, TE, Contact interactions on a lattice, Ann. Probab., 2, 969-988, (1974) · Zbl 0334.60052

[16] Harris, TE, Additive set-valued Markov processes and graphical methods, Ann. Probab., 6, 355-378, (1978) · Zbl 0378.60106

[17] Kuczek, T, The central limit theorem for the right edge of supercritical oriented percolation, Ann. Probab., 17, 1322-1332, (1989) · Zbl 0694.60092