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Avoiding effective packing dimension 1 below array noncomputable c.e. degrees. (English)

Zbl 1415.03047
J. Symb. Log. 83, No. 2, 717-739 (2018).

The present article is concerned with the notion of effective packing dimension when applied to Turing lower cones. *L. Bienvenu* et al. [Theory Comput. Syst. 45, No. 4, 740–755 (2009; Zbl 1183.68281)] and *L. Fortnow* et al. [Lect. Notes Comput. Sci. 4051, 335–345 (2006; Zbl 1223.68060)] have independently shown that when taking the supremum over the effective packing dimensions of all sets in the Turing lower cone of a set X, then that number must either be 0 or 1. This behaviour is notable as it is in stark contrast to that observed by *J. S. Miller* [Adv. Math. 226, No. 1, 373–384 (2011; Zbl 1214.03030)] for the closely related notion of effective Hausdorff dimension.

An obvious example for when the supremum takes the value 0 is if X is computable: then all sets in X's lower cone are also computable, and all of them have effective packing dimension 0. For the value 1, there are two possible cases: The supremum might be an attained maximum over the lower cone; or it might be an unattained supremum. Clearly, lower cones of the first type exist: if X is Martin-Löf random, X has effective packing dimension 1. *C. J. Conidis* [J. Symb. Log. 77, No. 2, 447–474 (2012; Zbl 1251.03047)] showed that lower cones of the second type exist as well.

A natural question was then to ask which lower cones exactly are of the second type. The authors precisely answer this question for lower cones that are below the halting problem. A result of *M. Kummer* [SIAM J. Comput. 25, No. 6, 1123–1143 (1996; Zbl 0859.03015)] suggest that array noncomputability may have a role to play in this particular case. And indeed, within the c.e. sets, the present article characterizes the array noncomputable sets as exactly those below which a lower cone of the second type can be found.

The result is proved using pruned clumpy trees, building on the notion of clumpy trees introduced by *R. Downey* and *N. Greenberg* [Inf. Process. Lett. 108, No. 5, 298–303 (2008; Zbl 1191.68304)].

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MSC:

03D32 Algorithmic randomness and dimension
03D28 Other Turing degree structures
68Q30 Algorithmic information theory (Kolmogorov complexity, etc.)

Cited in **1** Document

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Kolmogorov complexity; effective packing dimension; array noncomputability

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