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Improved localized radial basis function collocation method for multi-dimensional convection-dominated problems. (English) Zbl 1403.65106

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Summary: In this paper, the localized radial basis function collocation method (LRBFCM) is combined with the partial upwind scheme for solving convection-dominated fluid flow problems. The localization technique adopted in LRBFCM has shown to be effective in avoiding the well known ill-conditioning problem of traditional meshless collocation method with globally defined radial basis functions (RBFs). For convection-diffusion problems with dominated convection, stiffness in the form of boundary/interior layers and shock waves emerge as convection overwhelms diffusion. We show in this paper that these kinds of stiff problems can be well tackled by combining the LRBFCM with partial upwind scheme. For verification, several numerical examples are given to demonstrate that this scheme improves the LRBFCM in providing stable, accurate, and oscillation-free solutions to one- and two-dimensional Burger's equations with shock waves and singular perturbation problems with turning points and boundary layers.

MSC:

65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

Keywords:

localized radial basis function collocation method; convection-dominated; upwind scheme; Burger's equation; singular perturbation

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