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A stable nodal integration method with strain gradient for static and dynamic analysis of solid mechanics. (English) Zbl 1403.65069

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Summary: A stable nodal integration method with strain gradient (SNIM-SG) for curing the temporal instability of node-based smoothed finite element method (NS-FEM) is proposed for dynamic problems using linear triangular and tetrahedron element. In each smoothing domain, except for considering the smoothed strain into the calculation of potential energy functional as NS-FEM, a term related to strain gradient is taken into account as a stabilization term. The proposed SNIM-SG can achieve appropriate system stiffness in strain energy between FEM and NS-FEM solutions and obtains quite favorable results in elastic and dynamic analysis. The accuracy and stability of SNIM-SG solution are studied through detailed analyzes of benchmark cases and practical engineering problems. In elastic-static analysis, it is found that SNIM-SG can provide higher accuracy in displacement field than the reference approaches do. In free vibration analysis, the spurious non-zero energy modes can be eliminated effectively owing to the fact that SNIM-SG solution strengthens the original relatively soft NS-FEM, and SNIM-SG is confirmed to obtain fairly accurate natural frequency values in various examples. All in all, SNIM-SG cures the flaws of NS-FEM and enhances the dominant of nodal integration. Thus, the efficacy of the presented formulation in solving solid mechanics problems is well represented and clarified.

MSC:

- [65M60](#) Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
- [65M12](#) Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- [74S05](#) Finite element methods applied to problems in solid mechanics

Cited in **16** Documents

Keywords:

[nodal integration method](#); [stability](#); [strain gradient](#); [numerical methods](#); [linear triangular element](#); [linear tetrahedron element](#)

Software:

[XFEM](#)

Full Text: [DOI](#)

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