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**Small and large scale behavior of moments of Poisson cluster processes.** (English)

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Summary: Poisson cluster processes are special point processes that find use in modeling Internet traffic, neural spike trains, computer failure times and other real-life phenomena. The focus of this work is on the various moments and cumulants of Poisson cluster processes, and specifically on their behavior at small and large scales. Under suitable assumptions motivated by the multiscale behavior of Internet traffic, it is shown that all these various quantities satisfy scale free (scaling) relations at both small and large scales. Only some of these relations turn out to carry information about salient model parameters of interest, and consequently can be used in the inference of the scaling behavior of Poisson cluster processes. At large scales, the derived results complement those available in the literature on the distributional convergence of normalized Poisson cluster processes, and also bring forward a more practical interpretation of the so-called slow and fast growth regimes. Finally, the results are applied to a real data trace from Internet traffic.

**MSC:**

- 60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
- 60G18 Self-similar stochastic processes
- 60K30 Applications of queueing theory (congestion, allocation, storage, traffic, etc.)
- 60G22 Fractional processes, including fractional Brownian motion

**Keywords:**

Poisson cluster process; scaling; moments; cumulants; heavy tails; slow growth regime; fast growth regime; internet traffic modeling

**Full Text:** [DOI](#)

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