Rybakov, V. V.

Problems of admissibility and substitution, logical equations and restricted theories of free algebras. (English) Zbl 0691.03012

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[For the entire collection see Zbl 0676.00003.]

For the reviewer, interesting results have been proved in this paper. A positive solution to Friedman's 40th problem and a negative answer to Kusznetsov's question are described.

Let A_j, B be formulas in the language of the intuitionistic propositional calculus H. Let $p_1, ..., p_n$ be all letters occurring in these formulas and $x_1, ..., x_n$ distinct variables. Expressions of the form

$$(1) \quad A_1(x_1, ..., x_n), ..., A_m(x_1, ..., x_n)/B(x_1, ..., x_n)$$

are called rules of inference. The rule (1) is said to be admissible in H iff for all formulas $B_1, ..., B_n$, $A_j(B_1, ..., B_n) \in H$, $j = 1, ..., m$, imply $B(B_1, ..., B_n) \in H$. The rule (1) is called derivable in H if from $A_1, ..., A_m$ and the set of theorems of H one may derive B with the help of modus ponens. It is clear that derivability implies admissibility. Harrop's rule $(\neg p \supset (q \vee r))/(\neg p \supset q) \vee (\neg p \supset r)$ is an example of an admissible rule in H which is not derivable in H. The rule r is said to be a corollary of the rules $r_1, ..., r_n$ in H iff the consequence of r is derivable from the premisses of r with the help of a theorem of H, the rules $r_1, ..., r_n$ and modus ponens. A set B of admissible rules in H is called a basis for H if each admissible rule in H is the corollary of rules $r_1, ..., r_n$ in B. Friedman posed the problem of finding an algorithm which recognizes the admissibility of rules in H. Kuznetsov asked whether H has a finite basis for the admissible rules. There is shown that the problem of admissibility in H for rules has an algorithm and that H has no finite bases. Similar results are obtained for the modal logics S4 and Grz $(= S4 + \Box(\Box(p \to \Box p) \to p) \to p)$. In fact, a result for H is obtained from a result for S4.

Reviewer: Y.Komori

MSC:

03B55 Intermediate logics
03B45 Modal logic (including the logic of norms)
08B20 Free algebras
03B20 Subsystems of classical logic (including intuitionistic logic)

Keywords:

intuitionistic logic; free algebra; positive solution to Friedman's 40th problem; negative answer to Kusznetsov's question; intuitionistic propositional calculus; derivability; algorithm; admissibility of rules; finite basis; modal logics; S4; Grz