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On physical-constraints-preserving schemes for special relativistic magnetohydrodynamics with a general equation of state. (English) Zbl 1394.65149

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Summary: The paper studies the physical-constraints-preserving (PCP) schemes for multi-dimensional special relativistic magnetohydrodynamics with a general equation of state (EOS) on more general meshes. It is an extension of the work [*K. Wu* and *H. Tang*, Math. Models Methods Appl. Sci. 27, No. 10, 1871–1928 (2017; Zbl 1371.76096)] which focuses on the ideal EOS and uniform Cartesian meshes. The general EOS without a special expression poses some additional difficulties in discussing the mathematical properties of admissible state set with the physical constraints on the fluid velocity, density and pressure. Rigorous analyses are provided for the PCP property of finite volume or discontinuous Galerkin schemes with the Lax-Friedrichs (LxF)-type flux on a general mesh with non-self-intersecting polytopes. Those are built on a more general form of generalized LxF splitting property and a different convex decomposition technique. It is shown in theory that the PCP property is closely connected with a discrete divergence-free condition, which is proposed on the general mesh and milder than that in [Wu and Tang, loc. cit.].

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
76M10 Finite element methods applied to problems in fluid mechanics
76Y05 Quantum hydrodynamics and relativistic hydrodynamics
76W05 Magnetohydrodynamics and electrohydrodynamics

Keywords:

relativistic magnetohydrodynamics; equation of state; physical-constraints-preserving schemes; admissible state set; convexity; generalized Lax-Friedrichs splitting; discrete divergence-free condition

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References:

[1] Anderson, M; Hirschmann, EW; Liebling, SL; Neilsen, D, Relativistic MHD with adaptive mesh refinement, Class. Quantum Gravity, 23, 6503-6524, (2006) · Zbl 1133.83343

[2] Balsara, DS, Total variation diminishing scheme for relativistic magnetohydrodynamics, Astrophys. J. Suppl. Ser., 132, 83-101, (2001)

[3] Balsara, DS, Second-order-accurate schemes for magnetohydrodynamics with divergence-free reconstruction, Astrophys. J. Suppl. Ser., 151, 149-184, (2004)

[4] Balsara, DS, Divergence-free reconstruction of magnetic fields and WENO schemes for magnetohydrodynamics, J. Comput. Phys., 228, 5040-5056, (2009) · Zbl 1280.76030

[5] Balsara, DS, Self-adjusting, positivity preserving high order schemes for hydrodynamics and magnetohydrodynamics, J. Comput. Phys., 231, 7504-7517, (2012)

[6] Balsara, DS; Käppeli, R, Von Neumann stability analysis of globally divergence-free RKDG schemes for the induction equation using multidimensional Riemann solvers, J. Comput. Phys., 336, 104-127, (2017) · Zbl 1375.76212

[7] Balsara, DS; Kim, J, A subluminal relativistic magnetohydrodynamics scheme with ADER-WENO predictor and multidimensional Riemann solver-based corrector, J. Comput. Phys., 312, 357-384, (2016) · Zbl 1351.76157

[8] Brackbill, JU; Barnes, DC, The effect of nonzero $⊠ · \text bf{B} $ on the numerical solution of the magnetohydrodynamic equations, J. Comput. Phys., 35, 426-430, (1980) · Zbl 0429.76079

[9] Cheng, Y; Li, FY; Qiu, JX; Xu, LW, Positivity-preserving DG and central DG methods for ideal MHD equations, J. Comput. Phys., 238, 255-280, (2013) · Zbl 1286.76162

[10] Christlieb, AJ; Liu, Y; Tang, Q; Xu, ZF, Positivity-preserving finite difference weighted ENO schemes with constrained transport for ideal magnetohydrodynamic equations, SIAM J. Sci. Comput., 37, a1825-a1845, (2015) · Zbl 1329.76225

[11] Cockburn, B; Hou, S; Shu, C-W, The Runge-Kutta local projection discontinuous Galerkin finite element method for conservation laws IV: the multidimensional case, Math. Comput., 54, 545-581, (1990) · Zbl 0695.65066

[12] Zanna, L; Bucciantini, N; Londrillo, P, An efficient shock-capturing central-type scheme for multidimensional relativistic flows II. magnetohydrodynamics, Astron. Astrophys., 400, 397-413, (2003) · Zbl 1222.76122

[13] Du, J; Shu, C-W, Positivity-preserving high-order schemes for conservation laws on arbitrarily distributed point clouds with a simple WENO limiter, Int. J. Numer. Anal. Model., 15, 1-25, (2018)

[14] Evans, CR; Hawley, JF, Simulation of magnetohydrodynamic flows: a constrained transport method, Astrophys. J., 332, 659-677, (1988)

[15] Font, JA, Numerical hydrodynamics and magnetohydrodynamics in general relativity, Living Rev. Relativity, 11, 7, (2008) · Zbl 1166.83003

[16] Giacomazzo, B; Rezzolla, L, The exact solution of the Riemann problem in relativistic magnetohydrodynamics, J. Fluid Mech., 562, 223-259, (2006) · Zbl 1097.76073

[17] Gottlieb, S; Ketcheson, DJ; Shu, C-W, High order strong stability preserving time discretizations, J. Sci. Comput., 38, 251-289, (2009) · Zbl 1203.65135

[18] He, P; Tang, HZ, An adaptive moving mesh method for two-dimensional relativistic magnetohydrodynamics, Comput. Fluids, 60, 1-20, (2012) · Zbl 1365.76337

[19] Hu, XY; Adams, NA; Shu, C-W, Positivity-preserving method for high-order conservative schemes solving compressible Euler equations, J. Comput. Phys., 242, 169-180, (2013) · Zbl 1311.76088

[20] Honkkila, V; Janhunen, P, HLLC solver for ideal relativistic MHD, J. Comput. Phys., 223, 643-656, (2007) · Zbl 1111.76036

[21] Kim, J; Balsara, DS, A stable HLLC Riemann solver for relativistic magnetohydrodynamics, J. Comput. Phys., 270, 634-639, (2014) · Zbl 1349.76618

[22] Komissarov, SS, A Godunov-type scheme for relativistic magnetohydrodynamics, Mon. Not. R. Astron. Soc., 303, 343-366, (1999)

[23] Li, FY; Shu, C-W, Locally divergence-free discontinuous Galerkin methods for MHD equations, J. Sci. Comput., 22, 413-442, (2005) · Zbl 1123.76341

[24] Li, H., Xie, S., Zhang, X.: A bound-preserving high order compact finite difference scheme for scalar convection diffusion equations. Math. Comput. (2018). https://www.math.purdue.edu/~zhan1966/research/paper/compactFD.pdf. Accessed 16 May 2018

[25] Li, FY; Xu, LW; Yakovlev, S, Central discontinuous Galerkin methods for ideal MHD equations with the exactly divergence-free magnetic field, J. Comput. Phys., 230, 4828-4847, (2011) · Zbl 1416.76117

[26] Liang, C; Xu, Z, Parametrized maximum principle preserving flux limiters for high-order schemes solving multi-dimensional scalar hyperbolic conservation laws, J. Sci. Comput., 58, 41-60, (2014) · Zbl 1286.65102

[27] Mathews, WG, The hydromagnetic free expansion of a relativistic gas, Astrophys. J., 165, 147-164, (1971)

[28] Martí, JM; Müller, E, Grid-based methods in relativistic hydrodynamics and magnetohydrodynamics, Living Rev. Comput. Astrophys., 1, 3, (2015)

[29] Mignone, A; Bodo, G, An HLLC Riemann solver for relativistic flows-II. magnetohydrodynamics, Mon. Not. R. Astron. Soc., 368, 1040-1054, (2006)

[30] Mignone, A; Plewa, T; Bodo, G, The piecewise parabolic method for multidimensional relativistic fluid dynamics, Astrophys. J. Suppl. Ser., 160, 199-219, (2005)

[31] Newman, WI; Hamlin, ND, Primitive variable determination in conservative relativistic magnetohydrodynamic simulations, SIAM J. Sci. Comput., 36, b661-b683, (2014) · Zbl 1303.83002

[32] Noble, SC; Gammie, CF; McKinney, JC; Zanna, LD, Primitive variable solvers for conservative general relativistic magneto-hydrodynamics, Astrophys. J. Suppl. Ser., 641, 626-637, (2006)

[33] Qamar, S; Warnecke, G, A high-order kinetic flux-splitting method for the relativistic magnetohydrodynamics, J. Comput. Phys., 205, 182-204, (2005) · Zbl 1087.76090

[34] Qin, T; Shu, C-W; Yang, Y, Bound-preserving discontinuous Galerkin methods for relativistic hydrodynamics, J. Comput. Phys., 315, 323-347, (2016) · Zbl 1349.83037

[35] Radice, D; Rezzolla, L; Galeazzi, F, High-order fully general-relativistic hydrodynamics: new approaches and tests, Class. Quantum Grav., 31, 075012, (2014) · Zbl 1291.83092

[36] Rezzolla, L., Zanotti, O.: Relativistic Hydrodynamics. Oxford University Press, Oxford (2013) · Zbl 1297.76002

[37] Rossmanith, JA, An unstaggered, high-resolution constrained transport method for magnetohydrodynamic flows, SIAM J. Sci. Comput., 28, 1766-1797, (2006) · Zbl 1344.76092

[38] Ryu, D; Chattopadhyay, I; Choi, E, Equation of state in numerical relativistic hydrodynamics, Astrophys. J. Suppl. Ser., 166, 410-420, (2006)

[39] Tóth, G, The $⊠ · \text bf{B} = 0$ constraint in shock-capturing magnetohydrodynamics codes, J. Comput. Phys., 161, 605-652, (2000) · Zbl 0980.76051

[40] Holst, B; Keppens, R; Meliani, Z, A multidimensional grid-adaptive relativistic magnetofluid code, Comput. Phys. Comm., 179, 617-627, (2008) · Zbl 1197.76085

[41] Wu, K, Design of provably physical-constraint-preserving methods for general relativistic hydrodynamics, Phys. Rev. D, 95, 103001, (2017)

[42] Wu, K.: Positivity-preserving analysis of numerical schemes for ideal magnetohydrodynamics. SIAM J. Numer. Anal. (2018). arXiv:1802.02278 · Zbl 1391.76369

[43] Wu, K., Shu, C.-W.: Provably positive discontinuous Galerkin methods for multidimensional ideal magnetohydrodynamics. SIAM J. Sci. Comput. (2018). https://www.brown.edu/research/projects/scientific-computing/scientific-computing-research-reports · Zbl 1097.76073

[44] Wu, K; Tang, HZ, High-order accurate physical-constraints-preserving finite difference WENO schemes for special relativistic hydrodynamics, J. Comput. Phys., 298, 539-564, (2015) · Zbl 1349.76550

[45] Wu, K; Tang, HZ, Admissible states and physical-constraints-preserving schemes for relativistic magnetohydrodynamic equations, Math. Models Methods Appl. Sci., 27, 1871-1928, (2017) · Zbl 1371.76096

[46] Wu, K; Tang, HZ, Physical-constraint-preserving central discontinuous Galerkin methods for special relativistic hydrodynamics with a general equation of state, Astrophys. J. Suppl. Ser., 228, 3, (2017)

[47] Xing, Y; Zhang, X; Shu, C-W, Positivity-preserving high order well-balanced discontinuous Galerkin methods for the shallow water equations, Adv. Water Res., 33, 1476-1493, (2010)

[48] Xiong, T; Qiu, J-M; Xu, Z, Parametrized positivity preserving flux limiters for the high order finite difference WENO scheme solving compressible Euler equations, J. Sci. Comput., 67, 1066-1088, (2016) · Zbl 1383.76365

[49] Xu, Z, Parametrized maximum principle preserving flux limiters for high order schemes solving hyperbolic conservation laws: one-dimensional scalar problem, Math. Comput., 83, 2213-2238, (2014) · Zbl 1300.65063

[50] Xu, Z; Zhang, X; Abgrall, R (ed.); Shu, C-W (ed.), Bound-preserving high order schemes, No. 18, 81-102, (2017), Amsterdam · Zbl 1368.65149

[51] Yang, H; Li, FY, Stability analysis and error estimates of an exactly divergence-free method for the magnetic induction equations, ESAIM: Math. Model. Numer. Anal., 50, 965-993, (2016) · Zbl 1348.78028

[52] Zanotti, O; Fambri, F; Dumbser, M, Solving the relativistic magnetohydrodynamics equations with ADER discontinuous Galerkin methods, a posteriori subcell limiting and adaptive mesh refinement, Mon. Not. R. Astron. Soc., 452, 3010-3029, (2015)

[53] Zhang, X, On positivity-preserving high order discontinuous Galerkin schemes for compressible Navier-Stokes equations, J. Comput. Phys., 328, 301-343, (2017)

[54] Zhang, X; Shu, C-W, On maximum-principle-satisfying high order schemes for scalar conservation laws, J. Comput. Phys., 229, 3091-3120, (2010) · Zbl 1187.65096

[55] Zhang, X; Shu, C-W, On positivity-preserving high-order discontinuous Galerkin schemes for compressible Euler equations on rectangular meshes, J. Comput. Phys., 229, 8918-8934, (2010) · Zbl 1282.76128

[56] Zhang, X; Shu, C-W, Maximum-principle-satisfying and positivity-preserving high-order schemes for conservation laws: survey and new developments, Proc. R. Soc. A, 467, 2752-2776, (2011) · Zbl 1222.65107

[57] Zhang, X; Xia, Y; Shu, C-W, Maximum-principle-satisfying and positivity-preserving high order discontinuous Galerkin schemes for conservation laws on triangular meshes, J. Sci. Comput., 50, 29-62, (2012) · Zbl 1247.65131

[58] Zhao, J; Tang, HZ, Runge-Kutta discontinuous Galerkin methods for the special relativistic magnetohydrodynamics, J. Comput. Phys., 343, 33-72, (2017) · Zbl 1380.76048